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POWER DISTRIBUTION AND CURRENCY AREA PARTICIPATION

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In any cooperative agreement, voluntary participation will not occur unless all participants think the total decision-making power is distributed in a way that protects their individual interests. This paper examines the topic first from a game theoretic perspective, determining that highly asymmetrical power distributions are unsustainable when the choice variables of participants are strategic substitutes, but all power distributions can be sustained when the variables are strategic complements. The topic is then analyzed through a general equilibrium model which shows that small countries will not participate in a currency union unless they can assure their power in decision making processes is more than proportional to the size of their country.

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Table of Contents

I. Introduction

The feasibility and implications of currency unions became a major topic of discussion in international economics during the 1960s thanks to work done by Robert Mundell, Ronald McKinnon, and Peter Kenen. Subsequent research has investigated the factors that should be considered in creating an optimal currency area such as interest rates, labor mobility, economic shocks, and the openness of participating economies. One important factor that has not been significantly explored in the literature is the influence of the distribution of power amongst the currency union members. This paper's model of a currency union focuses on the share of the total decision making power that is distributed to each potential member. The interaction of nations in any international system is impacted by the balance of power across the involved states; therefore it is crucial to consider power distribution when determining an optimum currency union agreement.

Studying and understanding optimal power distribution in currency areas is important as the world addresses the current financial crisis, particularly the Sovereign Debt Crisis in the eurozone. Studies of the eurozone's creation and continued growth show that the euro countries do not, never have, and never will, make up an optimal currency area because they fail to meet many of the optimum currency area criteria. One suggested remedy is a split of the area into two or more currency groups. In order to do so successfully, the theoretical factors that determine the optimality of a currency union need to be fully understood and correctly implemented. Power distribution should be seriously considered in this case since the eurozone is made up of countries varying greatly in size and economic power. Understanding optimal currency areas is also important as other currency areas are considered, or even created, so that the mistakes made in creating the eurozone are not repeated in years to come.

The model is an expansion of a two-country model established in Alessandra Casella's 1992 work *Participation in a Currency Union*. Adding a third country to this model creates the opportunity to analyze how combinations of various sized countries will behave in a currency union, and the amount of relative power each requires to be willing to participate in either two or three country unions within the three country world. This expansion makes the new model more applicable than the two country model for addressing real-world situations since the countries are no longer restricted to either cooperating in a two country union or working independently. The three country model allows for the creation of a three country currency union or three different two country unions in addition to the national currency option where all nations maintain independence. This variety of situations opens the results to further analysis than is possible when only two countries are included and the possible outcomes are limited to a single currency union or independence for both nations.

The organization of the paper is as follows: Section 2 summarizes relevant literature on optimal currency areas and balance of power theories; Section 3 details an intuitive, game-theoretic model of a three country currency area; Section 4 describes the foundations of the general equilibrium model; Section 5 examines the model in which all three nations maintain distinct national currencies; Section 6 examines models in which either two or three of the involved nations share a common currency; Section 7 analyzes the willingness of countries to cooperatively form a currency union; Section 8 discusses real world applications of the model; and Section 9 concludes the paper and suggests future extensions to the model.

II. Literature Review

A. Optimal Currency Areas

The study of optimal currency areas began with Mundell's definition of a currency area as "a domain in which exchange rates are fixed." (1961, p. 657) At the time, few economists thought creating a currency area would be possible because countries were unwilling to sacrifice their national sovereignty. Due to this skepticism, Mundell ultimately concluded that the world was not yet politically prepared for currency unions, but his paper sparked decades of discussion on the topic of optimal currency unions. McKinnon (1963) expanded on Mundell's idea that optimum currency areas are regions based on similar industry by adding the concept of open economies, arguing that flexible exchange rates are least effective when economies are open to external trade. He claimed that an optimal currency area must be able to maintain full employment, balanced international payments, and a stable average price level. Kenen (1969) argued that regions with higher product diversification are better candidates for currency areas since, compared to areas with low product diversification, they are subject to smaller disturbances from economic shocks.

Tavlas (1993) built upon Kenen's work by creating the new theory of optimum currency areas based on inflation rates, factor mobility, openness of economies, commodity diversification, price and wage flexibility, goods market and fiscal integration, and political factors. Tavlas' theory and the core theories presented by Mundell, McKinnon, and Kenen were augmented by the creation of a general equilibrium model. (Bayoumi, 1994) Few other theoretical models have been created in this field because of the complexities involved, particularly due to the fact that politics tend to overshadow economic reasoning for or against currency unions. The theoretical models that have been designed often focus on a particular aspect of currency areas, such as the optimum number of currency areas or the importance of a particular optimal currency area criteria.[1] Numerous empirical studies have also been performed to analyze specific aspects of currency unions.[2]

B. Power Distribution

Balance of power theories are widely applied in studies of international relations, but are most commonly studied alongside conflict escalation and alliance formation. Zinnes (1967) examined previous approaches to power distribution and recapitulated the findings as seven major theories of balance of power. The first six are considered the traditional views, based on the common idea that a balance of power exists when power is distributed among all states such that no single state holds an "overwhelming" or "preponderant" amount of the system's total power. The seventh theory strays from this idea and instead describes balance of power as each state having equal sentiments, both positive and negative, toward every other state in the system.

Niou and Ordeshook (1986) applied the balance of power theories to international systems with the goal of obtaining two types of stability - system stability and resource stability. They found that there is no magic number of countries required for a system to achieve either or both of these types. The authors' analysis of three-country systems and n-country systems shows that neither group was more or less stable than the other. Simultaneously, Wagner (1986) studied balance of power from a game theoretic perspective, and concluded that there is in fact an optimum number of states. He studied two, three, four, and five member systems, and his findings show that, although any of these systems can be sustained, three actor systems are more

[1] Theoretical models and theory contributions can be found in Aizenman and Flood (1993), Frankel (1998), Alesina and Barro (2002).

[2] Studies on these topics and related issues include Bayoumi and Eichengreen (1994), Gosh and Wolf (1994 and 1996)

likely to be stable than those with two, four, or five members. Since the results of these studies vary so drastically, the model established here will determine the optimal distribution of power among the member states in the union rather than attempting to determine the optimal number of members for a currency union.

C. Power Distribution in Currency Areas

Alessandra Casella's *Participation in a Currency Union* serves as a foundation for this model. It is one of few studies to combine the fields of currency areas and power distribution and the most recent paper to create a theoretical model combining the two topics. Casella (1992) presents a general-equilibrium model in which the world consists of only two countries, one substantially larger than the other, that can decide to form a currency union or maintain independence. Her model focuses on the difference in country size to determine how much relative power the smaller country will require in order to decide in favor of the union. As expected, the model establishes that the smaller country will only decide to join the currency union when the agreement gives it a more than proportional share of power so that its interests are not lost in the union's decision making processes.

Previous findings in the literature are consistent with smaller nations having more than a proportional share of power in international organizations. This may result from the fact that the function of most international organizations is to serve the common interests of all member states, typically by providing some sort of public-good, thereby giving smaller members the opportunity to free-ride on the larger members in the organization. (Olson and Zeckhauser 1966) This issue is the basis for the sizing of countries in Casella's model. She focuses on a cooperative agreement between two unequally sized nations in order to analyze the amount of power the smaller nation would require to be assured its specific interests are not overrun by the larger nation. The same basic currency area issues were addressed in an early work by Casella and Feinstein (1988), in which they created a model where two countries share a common currency in order to analyze how monetary arrangements affect the optimum financing for a public good. This equilibrium model is not suitable for comparing the size of member countries, and was thus improved upon by the model created in Casella's subsequent work.

Issues of power distribution in international systems are addressed in models previous to that of Casella and Feinstein. One of these previous models, created by Roberto Chang (1991), focuses on the bargaining game between members of a union; other models (Krugman 1981, Dixit and Stiglitz 1977) focus instead on the interaction of two imperfectly competitive economies. It is this second type of model on which Casella's work is primarily based though the public-finance perspective of her model stems from a different strand of literature. She follows the approach established by Matthew Canzoneri and Carol Rogers (1990), but differs from them in her focus on the distribution of power rather than the overall sustainability of the agreement. In terms of power distribution in cooperative agreements, the model created by Casella is most closely linked to those established in previous literature on tariff wars in international trade, which studied the effect of differing country sizes on relative gains from cooperative trade agreements.[3]

The model established in this paper expands on Casella's two-country model of currency unions by introducing more countries to the equation, and analyzing the distribution of power among the various-sized nations as the number of currency union members increases. Before formulating the general-equilibrium model, the issue will be examined from a game-theoretic perspective so that the later results can be understood more intuitively.

[3] In particular, Johnson (1954), Mayer (1981), and Kennan and Reizman (1988).

III. Intuition

In a cooperative agreement where power is distributed according to size, small countries have very little control over the group's decision making processes, and are therefore unable to address their own specific interests within the agreement. Will smaller partners choose to deviate from the agreement if they believe their interests are not given enough weight in the decision making process? In a three country world, would two smaller nations be willing to work with a larger nation or would they rather work together to compete with the larger nation? The following relatively simple model provides intuition for the more general three country model developed in this paper.

Consider a partnership between two agents that has been formed in order to solve an externality. If one of the agents determines the outcome of the partnership single-handedly, the second agent may end up worse off than he would have been had the partnership never been formed. If participation in the agreement is voluntary, then it may be in the best interest of both agents to share power more equally, so that neither ends up worse off than he would have if he had chosen to act independently. Similarly, if two nations are trying to form a cooperative agreement, it may behoove the larger country to, in a sense, bribe the smaller country. The larger country can do this by accepting terms with a more egalitarian division of power than that which would result from distributing power based on country size. These results can more easily be seen through the two-player game outlined in Casella's *Participation in a Currency Union.* (1992)

The following figures depict the major results of the game. In both graphs of Figures 1, R_A and R_B are the reaction functions for agents A and B, $\bar{V}_A$ and $\bar{V}_B$ are isoprofit lines for each agent, point N is the Nash equilibrium, and all cooperative equilibria for all possible distributions of power lie along the contract curve connecting points A and B. All outcomes Pareto-superior to the Nash equilibria are found in the shaded area between the isoprofit curves. In the graph on the left, the actions taken by A and B are strategic substitutes; therefore spillovers are taken to be negative. Since the change in payoff depends on the sign of spillovers, player A's payoff falls along the reaction function R_A as the value of player B's action,z_B, rises and player B's payoff similarly falls along the reaction function R_B as player A's action, z_A, rises. The graph on the right has a similar structure but differs in the fact that the actions are now taken to be strategic complements, making the slopes of the reaction functions positive rather than negative.

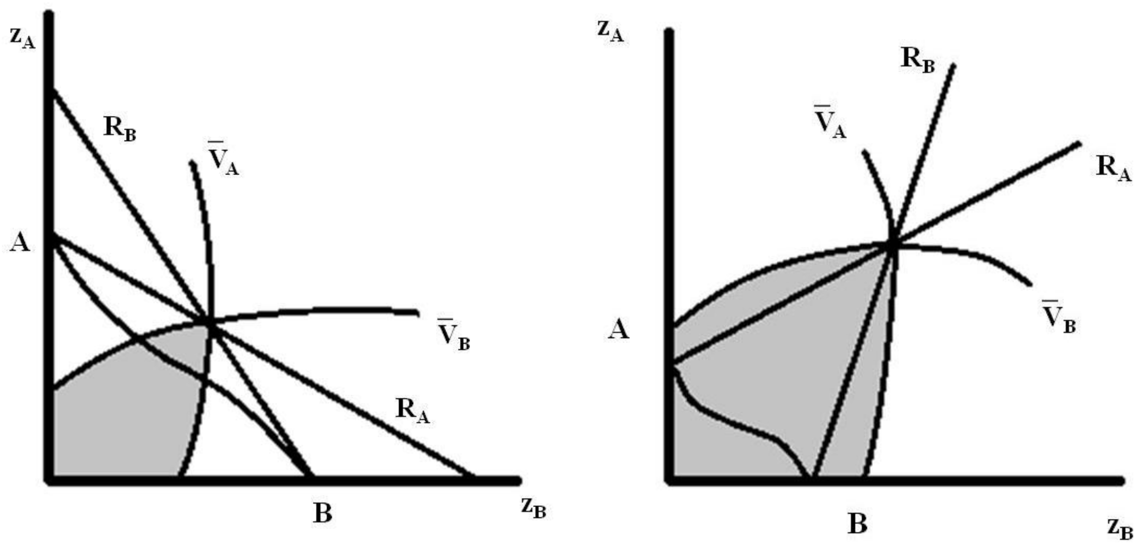

Figure 1: Actions as Strategic Substitutes (left) and Strategic Complements (right)

Comparing the two figures shows that all cooperative equilibria, and therefore all possible distributions of power, are Pareto-superior to the Nash equilibrium, N, only when the agents' actions are strategic complements, as in the graph on the right. When the actions taken by both agents are strategic substitutes, shown in the left graph, there are power levels that would produce outcomes where one player is worse off than he would be at the non-cooperative Nash equilibrium. This leads us to the conclusion that there is a minimum amount of power that must be allocated to the smaller nation in order to achieve sustainability when the agents' actions are strategic substitutes.

When one player controls the game, the outcome lies on that player's reaction function. If the actions are strategic substitutes than it is not possible for this outcome to fall within the range of Pareto optimal outcomes, but if the actions are strategic complements then the outcome will fall within this range. Movement along the reaction functions in the left graph results in a loss for the weaker agent, while movement along the reaction functions in the right graph benefits both agents. These results lead to the conclusion that negatively sloped reaction functions, as in the strategic substitutes case, lead to a breakdown of cooperation when power is distributed asymmetrically, making the weaker agent unwilling to participate in a cooperative agreement.

Much of this intuition can also be applied to a three-country situation, though the overall situation with three agents is undoubtedly more complicated. Possible cooperative agreements between three agents, A, B, and C, include a three-member agreement and three different two-member agreements: A and B, B and C, or A and C. The two-member agreements would follow the above model for those agents that have chosen to cooperate, and the remaining agent would simply act independently. This is likely to occur when two of the three agents can act as strategic complements, but the third is a strategic substitute.

Each of the three countries take an action, z_A , z_B , and z_C respectively, and their payoffs will be a function of all three of these actions and a parameter, σ_j where *j* = A, B, C.

$$V_j = V(z_A, z_B, z_C, \sigma_j) \tag{1}$$

All three actions must lie within a feasible range bound by 0 and z_A^{max} ,z_B^{max}, and z_C^{max} and the payoff functions will be strictly globally concave and twice continuously differentiable with spillovers that are everywhere finite and different from 0.

The Nash equilibrium for all three players (z_A^*, z_B^*, z_C^*) will be the intersection of the three reaction functions, implicitly defined as:

$$V_A^A(z_A, z_B, z_C, \sigma_A) = 0 \qquad (2a)$$

$$V_B^B(z_B, z_C, z_A, \sigma_B) = 0 \qquad (2b)$$

$$V_C^C(z_C, z_B, z_A, \sigma_C) = 0 \qquad (2c)$$

Since this equilibrium does not involve cooperation, it is not generally Pareto-optimal. Outcomes that are Pareto-optimal result from all three agents cooperating to maximize the jointly weighted sum of their payoffs

$$W = (3 - \gamma_B - \gamma_C)V_A(z_A, z_B, z_C, \sigma_A) + \gamma_B\, V_B(z_B, z_C, z_A, \sigma_B) + \gamma_C V_C(z_C, z_A, z_B, \sigma_C) \qquad (3)$$

The rest of the three-country intuition follows the same logic as the two-country example, since the three-dimension figures would simply be expansions of the two-dimension graphs seen above. The major difference that comes from adding a third reaction function is that the shaded range of outcomes that are Pareto-superior to the Nash equilibrium is now narrower than it was when bound by only two functions. Therefore, it follows that cooperative equilibria are less frequent and harder to come by when a third agent is included in the game, and it will continue to become even more difficult as additional agents are included.

Adding a third agent does not significantly change the model's intuition but does require an adjustment to the proposition establishing the existence of a minimum power level for the smaller nation. Now, rather than having a single minimum value for the weight assigned to each nation's payoff, there are two minimum values required to obtain and sustain cooperation when the actions of all three nations are strategic substitutes.

PROPOSITION: *If the actions z_A, z_B, and z_C are strategic substitute, such that $V_A^{ABC} \leq 0$, $V_B^{BCA} \leq 0$, and $V_C^{CAB} \leq 0$, then there exists a minimum value for γ_B, denoted $\bar{\gamma}_B$, and a minimum value for γ_C, denoted $\bar{\gamma}_C$, such that no cooperative agreement between A, B, and C is sustainable if either $\gamma_B < \bar{\gamma}_B$ or $\gamma_C < \bar{\gamma}_C$.*

If either of these γ values is below its minimum, the associated agent will not be willing to participate in the agreement and cooperation will breakdown. Maintaining a balance of power that satisfies two minimum conditions yields a smaller set of cooperative outcomes. As the number of countries in the model increases, the number of minimum conditions also increases, thereby making cooperation more difficult to achieve and sustain. Examination of the three country model shows that, when all three of the actions are strategic complements, all possible values of γ_B and γ_C produce cooperative equilibria that are Pareto-optimal to the Nash equilibrium; this will hold true for any number of agents as long as every one of them is a strategic complement to the rest.

If the countries are strategic substitutes rather than strategic complements, then not all values of γ will produce results that are Pareto-superior to the Nash equilibrium. If two of the countries are strategic complements and the third is a strategic substitute, the two complementary countries will be able to form a sustainable agreement quite easily but inclusion of the noncomplementary country would quickly make the agreement less beneficial to at least one of the complementary nations if not both. This shows that, even with three countries included in the model, a two-country agreement with an independent third country may be more effective than a three-country agreement. Three countries looking to form a cooperative agreement could choose to form a three-country union or one of three different two-country unions. As with any decision, the increased number of possibilities generated by adding a third country makes cooperation more difficult than it was with only two countries. Increasing the number of countries in the model beyond three will continue to increase the number of potential currency unions and therefore further complicate the decision making process. The intuition that minimal levels of power distribution do in fact exist for cooperative agreements, such as currency unions, will now be examined from a mathematical approach through the creation of a general-equilibrium model.

IV. General-Equilibrium Model Foundations

In this model, the world consists of 3 countries: A, B, and C. The countries can either act independently, cooperate to form a three-country currency union, or two countries can cooperate to form a union while the third acts independently. All of these possibilities are examined in detail in the following sections. The total world population is normalized to 3, where A is home to $3-\sigma_B-\sigma_C$ consumers and B and C each have σ_B and σ_C consumers, respectively. All of the consumers like variety in consumption of private goods and require a public good that is produced by their domestic government. The utility function of consumers in each nation is

$$U_j = (1-g)\cdot \ln\left(\sum_{i=1}^{n} c_{ij}^{\theta}\right)^{1/\theta} + g\cdot \ln(\Gamma_j) \tag{4}$$

where the parameter g (<1) represents the relative demand for the public good, n is the total number of varieties of the private good available for consumption, c_{ij} is the consumption of the i^{th} variety of the private good by consumers in country j, and Γ_j is the public good produced in country j, where j indexes the country. The parameter θ, which is bound by zero and one, plays a crucial role in this equation because 1/(1- θ) is the elasticity of substitution among the different varieties of the private good. This means that the economies approach perfect competition as θ approaches 1. If the countries' economies are perfectly competitive then no international trade will occur, therefore nullifying the opportunity for international cooperation.

All varieties of the private good, regardless of the country in which they are produced, share the same technology such that

$$l_i = \alpha + \beta x_i \quad i = 1, \dots, n \tag{5}$$

where l_i is the labor employed in the production of variety i and x_i is the quantity of the i^{th} variety produced. Production of each variety involves a fixed cost, α, which guarantees that each private firm will specialize in producing only one variety of the good. Market entry barriers are non-existent in the industry and, in equilibrium, each firm makes zero profits.

The government in each country produces a public good with simple constant returns-to-scale technology. Production of the private good is modeled as

$$\Gamma_j = l_{\Gamma j} \quad j = \mathrm{A, B, C} \tag{6}$$

where $l_{\Gamma j}$ is the amount of domestic labor employed in the production of country j's public good.

Each government finances its labor costs by printing money based on the formula

$$w_j l_{\Gamma_j} = M_j \tag{7}$$

where w_j is the nominal wage of country j and M_j represents issues of new money in country j. Combining equations (6) and (7), it becomes clear that new money injections in terms of nominal domestic wages, indicated by the variable m, are equal to the supply of the domestic public good.

$$m_j = \Gamma_j \tag{8}$$

The game proceeds as follows. Consumers live for two periods. In the first period they are employed by either a private firm or the government and receive salaries for their work; in the second period they use their disposable income to consume both private and public goods. Money is the only asset in the economy therefore it must be used in all transactions and real income is reduced by inflation. Private firms use their current revenues to pay out employee salaries and the government uses new issuances of money to finance its labor costs. Firms set prices in order to maximize profits while consumers plan consumption in order to maximize utility and each government sets money supplies with the goal of maximizing the discounted welfare of both present and future generations of its citizens.

Since technologies are identical across all private firms, and each firm specializes in the production of only one variety, all varieties produced in the same country will be sold at the same price. Each firm will set its price so as to equate marginal revenue and marginal cost where $\beta(w)$ is the marginal cost and $1/(1-\theta)$ is the elasticity of demand. This implies that the price is

$$p_j = \left(\frac{\beta}{\theta}\right) w_j \quad j = \text{A, B, C.} \tag{9}$$

The zero-profit condition of the model gives the scale of production as

$$p_j x_{ij} = w_j l_{ij} = w_j(\alpha + \beta x_{ij}) \tag{10}$$

where x_{ij} is the production of the i^{th} variety of the private good in country j and l_{ij} is the labor employed in this production. Substituting (5) and (9) into (10) gives

$$x_{ij} = \frac{\alpha\theta}{\beta(1-\theta)} = x \tag{11}$$

which shows that all varieties, regardless of the country in which they are produced, are produced in the same quantity since the quantity of each good is determined entirely by model parameters.

The consumer utility functions are structured such that consumers will spend the same amount on each variety of the private good available to them, regardless of which country the goods are produced in. If every good must be purchased with the currency of its domestic country, this implies

$$e_{BA} p_B x_B = p_A x_A \tag{12a}$$

$$e_{CA} p_C x_C = p_A x_A \tag{12b}$$

$$e_{CB} p_C x_C = p_B x_B \tag{12c}$$

where e_{jk} is the exchange rate between countries j and k defined as units of k currency per unit of j currency. Given (9) and (11), this yields

$$e_{BA} w_B = w_A \tag{13a}$$

$$e_{CA} w_C = w_A \tag{13b}$$

$$e_{\mathrm{CB}} w_C = w_{\mathrm{B}} \tag{13c}$$

$$e_{BA}p_B = \mathrm{p}_A \tag{14a}$$

$$e_{CA}p_C = p_A \tag{14b}$$

$$e_{\mathrm{CB}}p_C = \mathrm{p}_\mathrm{B}. \tag{14c}$$

Following the assumptions that technology is the same for all firms, regardless of country, and there are zero profits everywhere, both wages and prices will be equalized.

Flexibility of prices and wages insures full employment, such that:

$$n_A l_A = n_A(\alpha + \beta x) = 3 - \sigma_B - \sigma_c - l_{\Gamma\mathrm{A}} \tag{15a}$$

$$n_B l_\mathrm{B} = n_\mathrm{B}(\alpha + \beta x) = \sigma_B - l_{\Gamma\mathrm{B}} \tag{15b}$$

$$n_C l_\mathrm{C} = n_\mathrm{C}(\alpha + \beta x) = \sigma_C - l_{\Gamma\mathrm{C}} \tag{15c}$$

where n_j is the number of varieties of the private good produced in country j. Substituting (6) and (11) into (15a-c) gives the following equations for the number of private firms in each country.

$$n_A = \frac{(3-\sigma_B-\sigma_c-\Gamma_A)(1-\theta)}{\alpha} \tag{16a}$$

$$n_B = \frac{(\sigma_B-\Gamma_B)(1-\theta)}{\alpha} \tag{16b}$$

$$n_C = \frac{(\sigma_C-\Gamma_C)(1-\theta)}{\alpha} \tag{16c}$$

Since all varieties of the private good have equal prices and consumers disperse spending of their disposable income equally across all available varieties, the utility function of the current generation simplifies to

$$U_j = (1-g)\ln\left[(n_A + n_\mathrm{B} + n_\mathrm{C})c_j^{\theta}\right]^{1/\theta} + g \cdot ln(\Gamma_j) \tag{17}$$

where c_j is per capita consumption of each private good variety by consumers in country j and j indexes the countries.

Since consumption takes place in the second period of consumers' lives but their wages are earned in the first period, we find

$$c_A = \frac{\left(\frac{w_{\mathrm{A},-1}}{p_A}\right)}{(n_A+n_B+n_C)} \tag{18a}$$

$$c_B = \frac{\left(\frac{w_{\mathrm{B},-1}}{p_B}\right)}{(n_A+n_B+n_C)} \tag{18b}$$

$$c_C = \frac{\left(\frac{w_{\mathrm{C},-1}}{p_C}\right)}{(n_A+n_B+n_C)} \tag{18c}$$

where w_{-1} is the nominal wage paid in period -1, i.e. the period preceding consumption.

It is necessary to ensure that the production of each variety equals its total demand so that markets are in equilibrium. This is depicted by the following equation:

$$x = (3 - \sigma_B - \sigma_C)c_A + \sigma_B c_B + \sigma_c c_c \tag{19}$$

Once the monetary regime has been specified, the inflation rates of all three countries are determined as functions of each government's policies and it is then possible to express consumption in terms of the countries' money supplies and derive the indirect utility functions $U_A(m_A, m_B, m_C)$, $U_B(m_A, m_B, m_C)$, and $U_C(m_A, m_B, m_C)$. Since each government sets its money supply with the goal of maximizing the discounted welfare of both present and future generations of its citizens, the government problem for each nation can be written as

$$\max_{\mathrm{m_{j,t}}} \sum_{t=0}^{\infty} \delta^t U_{j,t}\left(m_{A,t}, m_{B,t}, m_{C,t}\right) \tag{20}$$

where δ, valued between 0 and 1, is the discount factor, t denotes the current period of the infinite-horizon repeated game, and j indexes the countries. Since this is an infinite-horizon repeated game, multiple equilibria are sustainable when appropriate punishment schemes are implemented but, for the purpose of this paper, the focus is on the simplest subgame-perfect equilibria in which the three governments repeat their optimal one-shot strategy in each period of the game and take foreign policy decisions as given. For the remainder of the paper, the objective function of the government policymaker for each nation is the welfare of a representative consumer in that nation and the aforementioned population parameters instead represent each country's general economic size. This per capita analysis produces the same conclusions that would be found if the analysis were conducted in aggregate terms instead.

V. National Currencies

If all three countries have their own national currency and each private good must be purchased with the national currency of the country in which it was produced, then international trade requires a market for foreign exchange. The equilibrium condition for the foreign exchange market is as follows:

$$[(\sigma_B c_B) + (\sigma_C c_C)]p_{\mathrm{A}} n_A = [(3 - \sigma_B - \sigma_C)c_A + (\sigma_C c_C)]p_{\mathrm{B}} n_B = [(3 - \sigma_B - \sigma_C)c_A + (\sigma_B c_B)]p_{\mathrm{C}} n_C \quad (21)$$

Total expenditures on products from country A by consumers in countries B and C must equal total expenditures on products from country B by consumers in countries A and C which must also be equal to the total expenditures on products from country C by consumers in countries A and B. This ensures that all three countries have an equal amount of their national currency in the foreign market.

Inflation rates are determined by the equilibrium conditions in the three domestic money markets which are implied by the equilibrium in both the goods market and foreign exchange market by Walras' law. Inflation rates can be obtained by recognizing that all monetary transactions inside a country take place in that country's domestic currency, therefore

$$(3 - \sigma_{\mathrm{B}} - \sigma_C)w_A = w_{A,-1}(3 - \sigma_{\mathrm{B}} - \sigma_C) + M_A \tag{22a}$$

$$\sigma_B w_B = w_{\mathrm{B},-1}\sigma_B + M_B \tag{22b}$$

$$\sigma_C w_{\mathrm{C}} = w_{\mathrm{C},-1}\sigma_C + M_C \tag{22c}$$

These equations can also be rewritten as

$$\frac{\mathrm{w}_A}{\mathrm{w}_{A,-1}} = \frac{3 - \sigma_B - \sigma_C}{3 - \sigma_B - \sigma_C - m_A} \tag{23a}$$

$$\frac{\mathrm{w}_B}{\mathrm{w}_{\mathrm{B},-1}} = \frac{\sigma_B}{\sigma_B - m_B} \tag{23b}$$

$$\frac{\mathrm{w}_C}{\mathrm{w}_{C,-1}} = \frac{\sigma_C}{\sigma_C - m_C} \tag{23c}$$

Recalling equation (13a-c) gives

$$\frac{e_{BA}}{\mathrm{e}_{BA,-1}} = \left(\frac{w_A}{w_{\mathrm{A},-1}}\right)\left(\frac{w_{B,-1}}{w_B}\right) \tag{24a}$$

$$\frac{e_{CA}}{\mathrm{e}_{CA,-1}} = \left(\frac{w_A}{w_{\mathrm{A},-1}}\right)\left(\frac{w_{C,-1}}{w_C}\right) \tag{24b}$$

$$\frac{e_{CB}}{\mathrm{e}_{CB.-1}} = \left(\frac{w_B}{w_{\mathrm{B},-1}}\right)\left(\frac{w_{C,-1}}{w_C}\right) \tag{24c}$$

Therefore it is clear that, in each country, inflation rates depend on the percentage of the domestic labor force employed by the government, since the salaries of government employees are paid with new issues of money, and the exchange rate subsequently changes to accommodate for the difference in inflation rates between countries.

Substituting (16a-c) and (23a-c) into (17) allows for per capita consumption of each private good variety to be written as

$$c_A = \frac{\alpha\theta(3-\sigma_B-\sigma_C-\Gamma_A)}{\beta(1-\theta)(3-\sigma_B-\sigma_C)(3-\Gamma_A-\Gamma_B-\Gamma_C)} \tag{25a}$$

$$c_B = \frac{\alpha\theta(\sigma_B-\Gamma_B)}{\beta(1-\theta)(\sigma_B)(3-\Gamma_A-\Gamma_B-\Gamma_C)} \tag{25b}$$

$$c_C = \frac{\alpha\theta(\sigma_C-\Gamma_C)}{\beta(1-\theta)(\sigma_C)(3-\Gamma_A-\Gamma_B-\Gamma_C)} \tag{25c}$$

The reliance on labor endowment, and therefore absence of policy variables, in this equation shows that the existence of national currencies insures that domestic purchasing power cannot be increased simply by issuing flat money. Define C_A , C_B , and C_C as total private consumption in each country, written in terms of labor units

$$C_A = c_A(3-\sigma_B-\sigma_C)\left[\frac{p_A}{w_A}n_A + \frac{p_B}{w_B}n_B + \frac{p_c}{w_C}n_C\right] \tag{26a}$$

$$C_B = c_B\sigma_B\left[\frac{p_A}{w_A}n_A + \frac{p_B}{w_B}n_B + \frac{p_c}{w_C}n_C\right] \tag{26b}$$

$$C_C = c_C\sigma_C\left[\frac{p_A}{w_A}n_A + \frac{p_B}{w_B}n_B + \frac{p_c}{w_C}n_C\right] \tag{26c}$$

Substituting (9), (16a-c), and (25a-c) into (26a-c) yields

$$C_A + \Gamma_A = 3-\sigma_B-\sigma_C \tag{27a}$$

$$C_B + \Gamma_B = \sigma_B \tag{27b}$$

$$C_C + \Gamma_C = \sigma_C \tag{27c}$$

Since money is the only asset in the economy and the labor supply is given, inflation tax is not able to affect any decision and is therefore not distortionary. Thus, as long as the exchange rates protect each country from foreign inflation rates, domestic money issues are equal to lump-sum taxes collected in the second period of each consumer's life.

Substituting (16a-c) and (25a-c) in (17) and recalling that the public good is equal to new issues of real money, the indirect utility functions of the current generation in each country are

$$U_A = K_A + \frac{(1-g)(1-\theta)}{\theta}\ln(3 - m_A - m_B - m_C) + (1-g)\ln(3 - \sigma_B - \sigma_C - m_A) + \mathrm{g}\ln(m_A) \quad (28a)$$

$$U_B = K_B + \frac{(1-g)(1-\theta)}{\theta}\ln(3 - m_A - m_B - m_C) + (1-g)\ln(\sigma_B - m_B) + \mathrm{g}\ln(m_B) \quad (28b)$$

$$U_C = K_C + \frac{(1-g)(1-\theta)}{\theta}\ln(3 - m_A - m_B - m_C) + (1-g)\ln(\sigma_C - m_C) + \mathrm{g}\ln(m_C) \quad (28c)$$

where

$$K_A = \frac{(1-g)(1-\theta)}{\theta} ln\left(\frac{1-\theta}{\alpha}\right) + (1-g)\ln\left(\frac{\theta}{\beta(3 - \sigma_B - \sigma_C)}\right) \quad (29a)$$

$$K_B = \frac{(1-g)(1-\theta)}{\theta} ln\left(\frac{1-\theta}{\alpha}\right) + (1-g)\ln\left(\frac{\theta}{\beta(\sigma_B)}\right) \quad (29b)$$

$$K_C = \frac{(1-g)(1-\theta)}{\theta} ln\left(\frac{1-\theta}{\alpha}\right) + (1-g)\ln\left(\frac{\theta}{\beta(\sigma_C)}\right) \quad (29c)$$

Each government maximizes the indirect utility of its current generation with respect to its own money supply, taking the foreign money supplies as given. The first-order conditions for countries A, B, and C respectively are

$$\frac{(1-g)(1-\theta)}{\theta(3 - m_A - m_B - m_C)} = \frac{g}{m_A} - \frac{1-g}{3 - \sigma_B - \sigma_C - m_A} \quad (30a)$$

$$\frac{(1-g)(1-\theta)}{\theta(3 - m_A - m_B - m_C)} = \frac{g}{m_B} - \frac{1-g}{\sigma_B - m_B} \quad (30b)$$

$$\frac{(1-g)(1-\theta)}{\theta(3-m_A-m_B-m_C)}=\frac{g}{m_C}-\frac{1-g}{\sigma_C-m_C} \tag{30c}$$

and can also be written as

$$\frac{g}{m_A}-\frac{1-g}{3-\sigma_B-\sigma_C-m_A}=\frac{g}{m_B}-\frac{1-g}{\sigma_B-m_\mathrm{B}}=\frac{g}{m_C}-\frac{1-g}{\sigma_C-m_C} \tag{31}$$

If μ is used to denote the share of resources devoted to the production of the public good within each country, (31) implies

$$(3-\sigma_B-\sigma_C)\left[\frac{g}{\mu_A}-\frac{1-g}{1-\mu_A}\right]=\sigma_B\left[\frac{g}{\mu_B}-\frac{1-g}{1-\mu_\mathrm{B}}\right]=\sigma_C\left[\frac{g}{\mu_C}-\frac{1-g}{1-\mu_C}\right] \tag{23}$$

Since the term inside the square brackets is a decreasing function of μ, when $3-\sigma_B-\sigma_C>\sigma_B>\sigma_C$, the values of μ must satisfy $\mu_A<\mu_B<\mu_C$ in order to maintain the equality of all three portions. This means that the smallest country always devotes the largest share of its total endowment to producing the public good and the largest country always devotes the smallest share of its endowment to the public good, when compared to the other nations involved.

Intuitively, it is reasonable for a country's size and the proportion of resources it devotes to public good production to be inversely related. Since the total number of varieties of private goods produced is based on the total amount of world resources, a small proportion of a larger country's endowment and a large proportion of a smaller country's endowment would have similar, if not equal, impact on the total amount of resources remaining for private good production. If all nations, regardless of size, devote the same proportion of their endowment to public good production, then larger nations diminish the total resources available to private goods with much larger impact than smaller nations do by devoting the same proportion of their resources to the public good.

VI. Common Currency Models

A. Three Countries, One Currency

When all countries share a common currency, the exchange rates, e_{BA}, e_{CA}, and e_{CB}, equal 1 in every period. Since the exchange rate is fixed, the inflation rates in all three nations will be equal. The constraint on the foreign-exchange market is now irrelevant because there are no international monetary accounts to clear if only one currency is in circulation. Nevertheless, all agents are bound by their budget constraints.

The common inflation rate for the three nations can be derived from the monetary equilibrium (21), taking into account that all transactions are carried out with the same currency rather than national currencies. Equation (33) models the wage market and implies the inflation rate depicted in equation (34).

$$3w = 3w_{-1} + M_A + M_B + M_C \tag{33}$$

$$\frac{w}{w_{-1}} = \frac{3}{3 - m_A - m_B - m_C} \tag{34}$$

The common inflation rate is determined by the total money injections of all three nations, relative to total world resources.

Per capita consumption of each variety of the private good, following the same methods as (25) in the national currencies model but using (34) in place of (23a-c), is found to be:

$$c_A = c_B = c_C = \frac{\alpha\theta}{3\beta(1-\theta)} \tag{35}$$

Since consumption is based purely on parameters that are equal across all involved nations, per capita consumption of each variety of the private good is the same regardless of the country in which the consumer lives. Substituting (11) into (35) gives the following simplified measure of per capita consumption.

$$c = \frac{x}{3} \tag{36}$$

Substituting (16a-c) and (35a-c) in (17) and recalling that the public good is equal to new issues of real money, each generation's utility is

$$U_A = \frac{(1-g)}{\theta}\ln(\frac{(1-\theta)}{\alpha}) + \frac{(1-g)}{\theta}\ln(3 - m_A - m_B - m_C) + (1-g)\ln\frac{(3-\sigma_B-\sigma_C)}{3} + \text{g}\ln(m_A) \quad \text{(37a)}$$

$$U_B = \frac{(1-g)}{\theta}\ln(\frac{(1-\theta)}{\alpha}) + \frac{(1-g)}{\theta}\ln(3 - m_A - m_B - m_C) + (1-g)\ln\frac{\sigma_B}{3} + \text{g}\ln(m_B) \quad \text{(37a)}$$

$$U_C = \frac{(1-g)}{\theta}\ln(\frac{(1-\theta)}{\alpha}) + \frac{(1-g)}{\theta}\ln(3 - m_A - m_B - m_C) + (1-g)\ln\frac{\sigma_C}{3} + \text{g}\ln(m_C) \quad \text{(37a)}$$

Suppose an international central bank is created to handle all monetary decisions for the currency union countries. The central bank sets money injections for each of the three nations so as to maximize a weighted sum of the above utility functions. The central bank's problem is modeled as

$$\max_{m_A, m_B} (3 - \gamma_B - \gamma_C)U_A(m_A, m_B, m_C) + \gamma_B U_B(m_A, m_B, m_C) + \gamma_C U_C(m_A, m_B, m_C) \tag{38}$$

constrained by $m_A \leq 3 - \sigma_B - \sigma_C$, $m_B \leq \sigma_B$, and $m_C \leq \sigma_C$, where utilities are as defined in (37a-c).

The first order conditions for the central bank's problem are as follows:

$$m_A = \min(3 - \sigma_B - \sigma_C, \frac{\Theta g(3 - \gamma_B - \gamma_C)}{1 - (1 - \Theta)g}) \quad (39a)$$

$$m_B = \min(\sigma_B, \frac{\Theta g \gamma_B}{1 - (1 - \Theta)g}) \quad (39b)$$

$$m_C = \min(\sigma_C, \frac{\Theta g \gamma_C}{1 - (1 - \Theta)g}) \quad (39c)$$

In these equations, γ_B and γ_C represent the share of the total currency union's power held by countries B and C, respectively. Likewise, γ_B and γ_C correspond to the amount of influence these two countries have over the currency union's central bank. If both γ_B and γ_C equal 1, then all three nations have an equal share of the power and the central bank gives equal weight to their utility functions when determining money supplies. Assigning equal weights to all three nations results in equal money supplies in all nations and therefore equal supplies of the public good.

If the central bank instead decides to choose γ_B and γ_C values based on σ_B and σ_C, then the utilities are weighted according to country size and the smallest country receives the smallest amount of funding for public good production while the largest country receives the largest amount of funding. This distribution maintains the initial levels of resources in all involved nations whereas equal money supplies across nations reduce the amount of resources in the largest nation by transferring them to the smaller nations. As explained in the national currencies model, equal proportions of public good financing, regardless of country size, are not efficient for the total world market. If all nations devote equal proportion of their endowment to public good production, then larger nations diminish the total resources available to private goods with much larger impact than smaller nations. Similarly, if the central bank gives all three nations equal weight and equal funding, the public good is overproduced in the smaller nations and will take away from the total world production of private goods.

Equal funding across nations means smaller nations are receiving more money per capita than larger nations, effectively increasing their share of world resources. This causes issues with inflation. Although money injections will generate inflation regardless of where the money is spent, the effect will be canceled out if money supplies are equal in per capita terms. Issuing money such that some countries have a higher per capita money supply than others increases the consumption of the public good in those countries more than it decreases the private disposable income of their consumers; creating a trade deficit that is financed by the seigniorage revenues generated by imbalanced money supplies.

Based on this information, the money supply each country receives in the currency union must be equal to its share of world resources in order to prevent transfers of resources between nations. Since money injections are determined by the share of power each nation holds in the central bank, power must also be distributed according to endowment in order to avoid transfers.

B. Three Countries, Two Currencies

Without loss of generality, suppose two of the three countries share a common currency, such that countries A and B are in a currency union while country C chooses to act independently. The exchange rate between countries A and B equals 1 in every period and the exchange rate between country C and either A or B is

$$\mathrm{e}p_C x_C = p_{AB} x_{AB} \tag{40}$$

By equations (9) and (11), the following are also true.

$$\mathrm{e}w_C = w_{\mathrm{AB}} \tag{41}$$

$$\mathrm{e}p_C = p_{\mathrm{AB}} \tag{42}$$

Since there are two currencies in circulation, international monetary accounts must be cleared. Equilibrium of the foreign exchange market is as follows:

$$(\sigma_C c_C)(p_{\mathrm{A}} n_A + p_{\mathrm{B}} n_B) = [(3 - \sigma_B - \sigma_C)c_A + (\sigma_B c_B)]p_{\mathrm{C}} n_C \tag{43}$$

The total amount spent by country C's consumers on goods from countries A and B must be equal to the total amount spent on goods from country C by consumers in A and B.

Since the exchange rate is fixed within a currency union, the inflation rates in countries A and B will be equal. The common inflation rate can be derived from the monetary equilibrium depicted in (43). Equation (44) models the wage market for the currency union and implies the inflation rate depicted in equation (45).

$$(3 - \sigma_C)w_{\mathrm{AB}} = (3 - \sigma_C)w_{\mathrm{AB},-1} + M_A + M_B \tag{44}$$

$$\frac{w_{AB}}{w_{AB,-1}} = \frac{(3-\sigma_C)}{3-\sigma_C-m_A-m_B} \tag{45}$$

Country C's wage market is described by

$$\sigma_C w_{\mathrm{C}} = \sigma_C w_{\mathrm{C},-1} + M_C \tag{46}$$

and implies the following inflation rate:

$$\frac{w_C}{w_{C,-1}} = \frac{\sigma_C}{\sigma_C - m_C} \tag{47}$$

Like equations (18a-c), consumption functions can be expressed in terms of the wages earned in the period prior to consumption, the domestic price, and the varieties of the private good available. Consumption functions are as follows:

$$c_A = c_B = \frac{\left(\frac{w_{\mathrm{AB},-1}}{p_{AB}}\right)}{(n_A + n_B + n_C)} \tag{48a}$$

$$c_C = \frac{\left(\frac{w_{\mathrm{C},-1}}{p_C}\right)}{(n_A + n_B + n_C)} \tag{48c}$$

Once again, it is necessary to ensure that the production of each variety equals its total demand so that markets are in equilibrium. Equation (19) is adjusted for the present situation; equilibrium demand is now written as

$$x = (3 - \sigma_C)c_{AB} + \sigma_c c_c \tag{49}$$

Per capita consumption of each variety of the private good can now be calculated by substituting (8), (9), (16a-c), (44), and (46) into (28a-b).

$$c_A = c_B = c_{AB} = \frac{\alpha\theta(3 - \sigma_C - m_A - m_B)}{\beta(1-\theta)(3-\sigma_C)(3 - m_A - m_B - m_{\mathrm{C}})} \tag{50a}$$

$$c_C = \frac{\alpha\theta(\sigma_C - m_C)}{\beta(1-\theta)(\sigma_C)(3 - m_A - m_B - m_{\mathrm{C}})} \tag{50b}$$

Substituting (16a-c) and (50a-b) in (17) and recalling that the public good is equal to new issues of real money, the indirect utility functions of the current generation in each country are

$$U_A = Q_{AB} + \frac{(1-g)(1-\theta)}{\theta}\ln(3 - m_A - m_B - m_C) + (1-g)\ln(3 - \sigma_C - m_A - m_B) + \mathrm{g}\ln(m_A) \quad \text{(51a)}$$

$$U_B = Q_{AB} + \frac{(1-g)(1-\theta)}{\theta}\ln(3 - m_A - m_B - m_C) + (1-g)\ln(3 - \sigma_C - m_A - m_B) + \mathrm{g}\ln(m_B) \quad \text{(51b)}$$

$$U_C = Q_C + \frac{(1-g)(1-\theta)}{\theta}\ln(3 - m_A - m_B - m_C) + (1-g)\ln(\sigma_C - m_C) + \mathrm{g}\ln(m_C) \quad \text{(51c)}$$

where

$$Q_{AB} = \frac{(1-g)(1-\theta)}{\theta} ln\left(\frac{1-\theta}{\alpha}\right) + (1-g)\ln\left(\frac{\theta}{\beta(3-\sigma_C)}\right) \quad \text{(52a)}$$

$$Q_C = \frac{(1-g)(1-\theta)}{\theta} ln\left(\frac{1-\theta}{\alpha}\right) + (1-g)\ln\left(\frac{\theta}{\beta(\sigma_C)}\right) \quad \text{(52b)}$$

Like the three-country currency union model, a central bank is formed to handle all monetary decisions for countries A and B. The central bank sets money injections for each of the two nations so as to maximize a weighted sum of their utility functions. The central bank's problem is modeled as

$$\max_{m_A, m_B} (2-\gamma_B)U_A(m_A, m_B, m_C) + \gamma_B U_B(m_A, m_B, m_C) \quad \text{(53)}$$

constrained by $m_A \leq 3 - \sigma_B - \sigma_C$ and $m_B \leq \sigma_B$.

Since country C is not included in the currency union, its money supply is not set by the central bank. Instead, the government of country C aims to maximize the discounted welfare of both present and future generations of its citizens in the same way as the government problem described previously. Country C's government's problem for this case can be written as

$$\max_{m_{C,t}} \sum_{t=0}^{\infty} \delta^t U_{C,t}(m_{A,t}, m_{B,t}, m_{C,t}) \tag{54}$$

The first order conditions for A and B follow the methods of the three-member currency union model. In order to obtain a closed form solution, we impose the constraint $m_C = \sigma_C$. This constraint implies that the central bank, in order to make decisions regarding the money supplies of A and B, assumes that country C will choose to set its money supply equal to its endowment in order to maximize the welfare of its citizens. Imposing this constraint results in the following first order conditions for countries A and B within the currency union:

$$m_A = \min(3 - \sigma_B - \sigma_C, \frac{\Theta g(\gamma\sigma_C - 3\gamma - 2\sigma_C + 6)}{-2(1 - (1 - \Theta)g)}) \tag{55a}$$

$$m_B = \min(\sigma_B, \frac{\Theta g\gamma(-3 + \sigma_C)}{-2(1 - (1 - \Theta)g)}) \tag{55b}$$

Country C’s first order condition can be obtained using the same methods used in the national currency model and is as follows:

$$\frac{(1 - g)(1 - \theta)}{\theta(3 - m_A - m_B - m_C)} = \frac{g}{m_C} - \frac{1 - g}{\sigma_C - m_C} \tag{56}$$

Since country C acts independently, it has full control over its own money supply. Country C’s government therefore has the ability to adjust its own money supply based on the decisions made by the currency union and will be able to maximize its citizens’ utility more efficiently. Countries A and B do not have the ability to adjust their own money supply and must rely on their central bank to determine their public good funding levels and, indirectly, the utilities of their citizens.

As shown by the central bank modeled in the three countries/one currency model explained above, central banks most effectively distribute money and avoid transfers between nations when each nation's money supply is based purely on its economic endowment. In this model, the first order conditions for countries A and B show that each country's money supply is equal to the minimum value between its endowment and a function of both model parameters and the world endowment outside of the currency union. When theoretically reasonable parameters are used to evaluate the first order condition functions, the results are consistently smaller than the country's endowment. Therefore, the money supply assigned to each nation participating in this model's currency union will be less than that country's endowment, making the distribution of money less optimal for currency union members here than in both the national currency model and the three-country currency union model.

VII. Currency Union Participation

Suppose all three nations are free to decide between joining the currency union and maintaining independence and therefore control of monetary policies. Although all three nations have this choice, two or more must decide to give up their independence in order for a union to be formed. Due to the lack of a closed-form solution for the equilibrium money supplies in the national currencies model, analytical expressions for minimum power distributions cannot be calculated. The following examination of the models uses multiple sets of theoretically realistic parameter values to analyze the open-form solutions.

When one of the nations is significantly smaller than the others, it will not be willing to sacrifice its independence unless it is guaranteed a share of the total power larger than it would have received if power was determined strictly by size. If the small nation accepts a level of power equal to its endowment, it will suffer the costs of coordination without reaping enough benefits to break even. This conclusion can be seen more clearly by analyzing the public good production in both the national currencies and common currency models. In the national currencies model, we saw that the smallest country allocates the largest share of its endowment to public good production when compared to the amounts allocated by larger nations. Thus, the per capita supply of the public good is largest in the smallest country. With the common currency model, in order to maintain resource levels, the central bank must assign weights based on country size. In this situation, the nations receive public good funding that is equal in per capita terms. Therefore, the smaller nation's consumers no longer have access to as much of the public good as they did under the national currencies model and the nation bears the brunt of the cooperation costs.

Although closed-form solutions do not exist for this model, the following graph depicts the minimum weight required for cooperation compared to the economic size of the nation in a typical case with set parameter values. Changing the parameter values adjusts the tightness of the curve to the 45° baseline but the overall results are not significantly impacted.

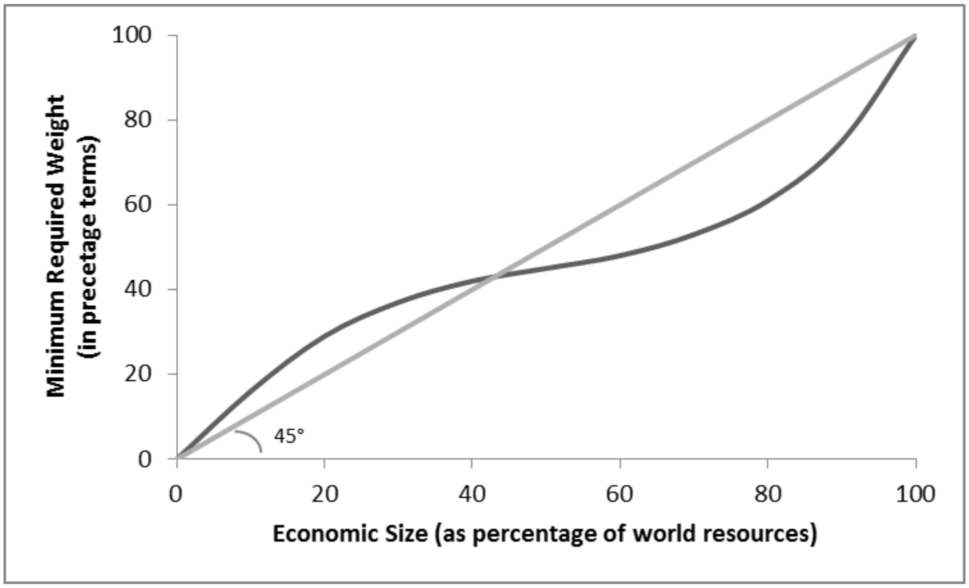

Figure 2: Economic Size and Minimum Percentage Weight in the Central Bank

As predicted in the intuition model, a relatively small nation will demand a share of power larger than its proportional endowment in order to participate in the union cooperatively. Figure 2 shows that this fact is not only true for countries that are significantly smaller than their counterparts, but also for countries that have as much as 40 percent of the total world endowment. This can be seen by comparing the curve of minimum γ values to the baseline of angle 45°. When the curve is above the baseline, the country requires a proportion of power larger than its endowment; when the curve lies below the baseline, the country is willing to accept a power level less than its endowment. The intersection of the curve and baseline establishes a turning point at 40% indicating that countries below this threshold are not willing to participate unless they are given more than their share of power. The curve to the right of this intersection confirms that smaller countries will in fact be able to obtain their desired level of power since larger countries are willing to participate in the agreement when given less power than their endowment implies.

A large country in a cooperative agreement can reduce its portion of the total power and still benefit from cooperating; therefore large nations are willing to bribe smaller nations with excess power to form cooperative agreements. Since any cooperative agreement results in less public good funding for smaller nations than they would have allocated under flexible exchange rates, a larger percentage of total world resources is devoted to private good production which benefits all involved nations compared to the purely domestic benefits of public good production. Since the larger country benefits from this increase in the production of private goods, it is willing to cede some power to the smaller country in order to ensure the formation of the cooperative agreement.

When only two countries are included in the currency union, power can easily be distributed so that the smaller country's demands are met without negatively impacting the larger country. Introducing a third country to the model makes this situation more difficult to address. Since at least two of the countries will have to fall below the 40% threshold established above, the demand for excess power will exceed the amount of power the largest country is willing to forfeit. Consider the case where all three nations are the same size. Each country holds 33% of the total resources but, because the curve is above the 45° baseline at that point, will demand more than 33% of the total power. If each of the three nations demands more than a third of the total power, there is not enough power to satisfy all members and the agreement is not sustainable. As the number of countries in the game increases, the power discrepancy will also increase and cooperative agreement will be impossible. Note, however, that this is simply the static game and that punishment schemes could be implemented to ease these issues.

VIII. Real World Applications

One important real world situation this model can easily be compared to is the eurozone. The European Central Bank (ECB) is the governing body for all monetary decisions in the 17 European nations that have adopted the euro since 1999. The bank's decision making council is made up of the governors of the 17 national central banks and the 6 members of the ECB Executive Board. Clearly, all 17 nations are represented in the agreement, but further examination shows that their influence on decisions is far from equal. Many of the bank's decisions come directly from the Executive Board, whose members are often representatives of the euro countries with the largest economic endowments. Based on the model established in this paper, the fact that the power is distributed roughly according to endowment means that transfers between nations is unlikely to occur simply through the setting of money supplies. Power distribution according to size has also been shown to result in unstable cooperative agreements, as is the case with the eurozone.

The situation of the eurozone is further complicated by the organization's push for continued expansion. Latvia has already been approved to adopt the Euro in January of 2014, Lithuania is on track to adopt the currency in 2015, and as many as six other nations could be integrated into the common currency over the next decade. If this expansion occurs without any other major changes to the eurozone, the union could have as many as 25 members, varying in size from Germany, which represents almost 27% of the eurozone's total GDP, to Malta, which is currently responsible for less than one tenth of a percent of the eurozone's total GDP. This large variance in size has already proved to be an issue for the union and increasing the variety of members will only make the issue worse.[4]

IX. Conclusion and Future Extensions to the Model

Currency Union participation has been shown to be beneficial from the perspective of participants only when each country has a substantial impact on the decisions made within the union. A small country will not be willing to participate in a currency union, or any other cooperative agreement, if it is not given a share of power that is larger than its proportional share of resources. Although punishment schemes may be implemented to counteract this demand, a truly cooperative agreement cannot be formed unless there is a disproportional distribution of power among the member states.

Economically, however, a disproportional distribution of power is not effective in terms of total world resources and the allocation of these resources to public and private good production. When money supply directly corresponds to domestic public good production, the per capita money supplies must be equal regardless of each nation's size so that no single nation diminishes the production of private goods more than any other nation. Therefore, the most economically efficient currency unions are those in which power is distributed according to size, even if the countries are not necessarily satisfied with the arrangement in terms of power distribution.

Although this model is already an extension of a simple two country general equilibrium model, it could be expanded further still to include more than three nations. Ideally, a model in

[4] These and further eurozone statistics can be found at http://epp.eurostat.ec.europa.eu/portal/page/portal/eurostat/home/

which the number of participating nations is limitless would be created so that it could be applied more effectively to discussions of current and potential currency unions around the world. A model of this form would allow for examination of situations in which more than one currency union could form simultaneously. This would provide insight in to how currency unions interact with one another in comparison to how countries interact with other countries or a single currency union. Further analysis of the open-formed solutions of this model without expanding beyond three countries may also prove beneficial in gaining a deeper understanding of the issues surrounding power distribution in currency areas on a smaller scale.

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