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COALITION BUILDING AND SUPERMAJORITIES

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Prior research suggests that when forming a voting coalition, supermajorities are more efficient then minimal winning coalitions. Due to a large amount of exogenous variables in previous literature's data sets, the results of coalition formation have not been measured accurately.

This paper addresses the shortcoming of previous literature by observing a closed game with limited exogenous variables. The outcomes of this game are known to be coalition built, so the results of this paper are representative of a coalition built voting structure.

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I. Coalition Formation

Voting is one of the oldest tools used to guide public decisions. Some of the earliest accounts of voting date back to 480 B.C. in Athens, and is then seen throughout the history of the world. Today many democratic countries function with voting as a primary basis for almost all decisions. In the United States, voting determines what laws are made, in Congress; and how the laws are interpreted, in the Supreme Court. It does not end there; voting is seen in the private sector as well. Corporations base their most important decisions on votes of the stockholders and board members.

With voting being such a crucial aspect to society, it is not surprising that there is a vast amount of literature, in both public economics and political science, attempting to understand the basics of how voting works. This paper aims to expand the understanding of how voting operates.

This paper will attempt to carve a small sliver into the understanding of how voting works. More precisely, this paper will focus on coalition formation within finite voting structures. In this paper, a finite voting structure refers to any voting situation where the number of voters is known with certainty. The Supreme Court is an example of a finite voting structure; as there are always nine voters. The boards of directors for corporations are also finite voting structures, as there are a set number of board members that go into each vote and this number is known to all of the people involved in the vote. Because of this knowledge of the amount of voters in the structure, it becomes very easy to calculate the number of votes needed to win any vote passed through a finite structure. For instance, in a vote of nine people, every member would know that a vote of five or greater is a winning vote. Due to this knowledge, it creates a strategic situation for any member attempting to win the vote for they know the exact number of people they need to bribe or persuade in order to win the vote.

In finite voting structures, each side of the vote attempts to build a coalition that is large enough to ensure a win for their side. Coalitions form through several different means; the two most prominent are bribery and logrolling. Bribery is when one a member pays other members to vote a certain way, while logrolling is when members trade votes. Bribery is a frowned upon method so it is less often seen then logrolling, however, it is useful to observe some theories in terms of bribery, because monetary values can be used in the discussion. Logrolling works very similar to bribery (Wiseman 2004), the theories for one can shed light upon how the other works. Since there are different methods that can be used to form coalitions, the theories of how they operate intertwine; this paper will refer to all methods of coalition formation as Coalition Building Efforts (CBE) to simplify the discussion.

One difficulty in analyzing coalition formation, however, is that the actual coalition forming is often unobservable. Only the outcome or vote is empirically observed, and the reasoning for these outcomes can most often only be hypothesized.

This study reviews theories of coalition building, then tests voting structures to see if these theories hold true. To ensure the outcomes are representative of coalition formation, the voting structure used in this paper will have known coalition building present. These results shed light upon what coalition formation looks like. These results also allow for observation of other finite voting structures, to see if outcomes represent themselves similarly. Structures that do represent similar voting outcomes may then be suspect of coalition formation as well.

II. Literature Review

Early theory on voting structures predicts that coalition formation should be minimal winning. (Baron and Ferejohn 1989; Riker 1962). Voting theory implies that coalitions form through coalition builders exerting CBE to pay voters, where the goal is to create the least expensive voting coalition. The theories presented in the papers by Baron and Riker form the same conclusions. When forming a voting coalition, the least costly method is to create a voting group that is the smallest size possible to win the vote. The logic behind these theories is that a smaller coalition means less people, and less people would mean less CBE. Contrary to these theories, empirics show that voting structures often form in larger groups than minimal winning, or what is referred to as a supermajority (Browne 1973; Lutz and Murry 1975).

The difference between a minimal winning coalition and a supermajority is the size of the coalition. A minimum winning coalition is any coalition that uses the smallest number of people possible to ensure the vote. For instance, if a vote takes place with nine voters, an outcome of four to five contains a minimal winning coalition. The coalition of five voters is minimal winning, because if any single voter left that coalition, then it would no longer be winning, so it is the smallest possible coalition size that could win the vote. A supermajority, however, is any voting size that is larger than minimal winning. In that same structure of nine voters, an outcome of three to six, or two to seven would be a supermajority, because the winning coalition contains more than the required number of voters needed to win.

Initial defense for supermajorities suggest that supermajority formation is present due to of the risk involved in a minimum winning coalition. A supermajority forms only when the coalition leader is uncertain if all members will act as they have promised (Koehler 1972). Other theories suggest that minimum winning coalitions are formed, but the medium voters' beliefs gravitate toward those of the existing majority, creating supermajority outcomes (Wiseman and Wright 2008). This gravitation is attributed to the general attitude of the voting group is supporting the majority's vote, so indifferent voters go along with the majority. This is more prominent as the number of voters in the structure increases because there are more undecided voters.

Groseclose and Snyder (1996) suggest that early theories explaining supermajorities are inaccurate. Groseclose claims that supermajorities form because they are the economically efficient coalition. The theory states that supermajorities form because they are less expensive to form then a minimum winning coalition. Groseclose's model has two opposing forces, the coalition builder (Player 1) and an opposition (Player 2). An example of the model can be seen in Table 1 below. In the game, Player 1 has the goal of building a winning coalition using the lowest amount of CBE possible; however, Player 2's goal is to use his available CBE to make Player 1's coalition a minority. Player 1 is aware of the CBE Player 2 has to spend, and Player 1 uses that amount to calculate how much CBE he must exert on each player. In Table 1, net values that are in brackets are the players that Player 2 must pay in order to achieve his goal. So in each scenario, the sum of the bracketed numbers is equal to the Player 2's total, which is 48. In order to further illustrate Groseclose's theory, this example is designed so that the initial preference the voters have toward to vote would lead to Player 2's victory, as only four voters have positive initial values, while the other five are negative.

Table 1. Example of Groseclose Model

Representation of Groseclose Thoery of Supermajorities
Opposition CBE=48

		Minimum Winning		Minimum+1		Minimum+2		Minimum+3	
Voter	Initial Value	CBE	Net Value	CBE	Net Value	CBE	Net Value	CBE	Net Value
1	19	29	48	5	24	0	19	0	19
2	14	34	48	10	24	2	16	0	14
3	9	39	48	15	24	7	16	3	12
4	4	44	48	20	24	12	16	8	12
5	-1	49	(48)	25	(24)	17	(16)	13	(12)
6	-6	0		30	(24)	22	(16)	18	(12)
7	-11	0		0		27	(16)	23	(12)
8	-16	0		0		0		28	(12)
9	-21	0		0		0		0	
Total CBE		195		105		87		93	

In this example, Player 1 should build a coalition of six voters because it is the least expensive coalition to build. A coalition of six has two more voters than a minimal winning coalition, and is a supermajority.

In real voting structures, however, the preferences of the individual voters are not usually known. The early voting theories on supermajorities suggest that this uncertainty would also be the cause of supermajorities (Koehler 1972). Contrary to this theory, Hummel (2009) claims that uncertainty of voter preferences should lower the size of the voting coalition. Hummel states that the coalition should represent Riker's minimum winning coalition model, because with uncertainty more CBE would be exerted per individual voter in order to ensure each voter exceeds their minimum threshold to vote with the coalition; making the minimum winning model less costly than a supermajority.

A study of logrolling as the method of CBE used to build supermajority coalitions exists within the US Senate (Lee 2000). In the Senate, the larger states have the same value of votes as less populated states, however, they receive a much larger benefit when a bill passes that favors their state. Lee claims that a larger state often supports bills for smaller states in order to form coalitions of smaller states. These coalitions of smaller states support the occasional bill that benefits that larger state in return.

Lee's conclusions are theoretical based of observation of the senate, as it is difficult to form an empirical analysis on voting structures. Most voting structures have many exogenous variables affecting the voting process, making any results subject to scrutiny. An attempt to analyze voting structures empirically usually involves tests such as three-stage least squares regression to treat the exogenous variables (Carraba and Volden 2004; Pereira and others 2009). Wiseman (2004) attempts to use an ordinary least squares regression to analyze legislators, but admits the model suffers largely from omitted variable bias.

It is of the interest of this paper to contribute to previous literature by constructing a test of voting structures in a setting with limited exogenous variables in an attempt to minimize error in the findings.

III. Data

The data set used is from the televised game *Survivor*. This data set is chosen for two main reasons: The benefit that the game is isolated from outside variables, and the strategic nature of the game. The game *Survivor* takes place in a remote location isolated from all forces outside of the game. The players are restricted from contact with any person or information source that is not a member of the game. The players also have no relation or contact with each other before the game begins. The isolation of the players is beneficial to this study, because it cuts out many exogenous variables that need to be considered in the statistical study.

The game also operates in a manner that creates a strategic situation that leads to coalition building. By using this game, the results are representative of coalition built voting situations. The game begins with 16-20 players, divided into two groups. Every three days, one group must vote to eliminate a member in that group. After approximately half of the members have been eliminated, the remaining players in the two groups merge together and continue in the same manner. A model of this game can be seen in the appendix.

The structure of the game gives incentives to each player to attempt to form a coalition that is large enough to ensure their own personal safety. The CBE in these coalitions is the amount of safety the coalition provides to each member.

Coalition building itself is not observable for the purposes of this study; however, with the data set known to be driven by coalition building, this paper can observe the voting outcomes influenced by the coalition building and shed light upon how votes present themselves in a coalition built setting.

This paper will refer to two separate levels of testing: The group level, and the individual level. The group level means the outcome of the vote will be considered as a single data point. And the individual level will use each vote cast as a data point. Each point will be given binary coding:

All Variables = 1 when yes

All Variables = 0 when no

Table 2. Group Level Variables

Y_1	Is there a supermajority in the current instance?
X_1	Was there a supermajority in the prior instance?
X_2	Has the merge occurred?

Table 3. Individual Level Variables

Y_2	Is the player in a supermajority in the current instance?
W_1	Was the player in a supermajority the prior instance?
W_2	Was the player in the majority the instance before?
W_3	Has the merge occurred?

This study will attempt to view supermajorities in multiple ways in order to form a complete view of how supermajorities present themselves in a coalition built setting. This study will observe if supermajorities have path dependency, and if supermajorities are responsive to a shock in the system.

Path dependency will be determined by observing if a supermajority affects the chance of a supermajority forming in the next voting instance. At the group level a logit procedure will regress:

$$Y_1 = \beta_1 X_1 + \alpha \tag{1}$$

To test if a supermajority in the instance before has an effect on whether a supermajority will occur in the current instance.

At the individual level two logit procedures are conducted to see if a player's participation in a supermajority is influenced by that player being in a supermajority or a majority in the instance before. This will be tested with the following two tests:

$$Y_2 = \beta_1 W_1 + \alpha \quad (2)$$

$$Y_2 = \beta_1 W_2 + \alpha \quad (3)$$

Test (2) observes the correlation of a player's choice to be in a supermajority with the decision to be in one the instance prior. And test (3) observes the relationship of being in a supermajority with being in a majority the instance prior. Since, W_1 is a subset of W_2 the results of test (3) are expected to carry a higher coefficient and have greater confidence, however, the results of test (3) carry slightly less economic meaning. Test (3) is used because these tests are designed to capture the individual decision making, regarding supermajorities. The individual cannot decide if there is a supermajority in each turn, but only what coalition they will join. So it is reasonable that a player who is in a majority in one instance and a supermajority in the next still has bearing on path dependency, since the player did not have a choice of being part of a supermajority in the prior instance.

The other factor that will be observed is if supermajorities are responsive to a shock in the system. The shock in the system here is the merge that occurs in each season. At the group level, two sample test of proportions will be performed to see if there is a statistical difference between the presence of supermajorities before and after the merge. The two samples will consist of the data points from Y_1 divided into two groups. Instances before a merge Y_{1x}, and instances after a merge Y_{1y}. The test will be:

H_o: Y_{1x}=Y_{1y} (4)

H_a: Y_{1x}>Y_{1y}

The test will be one tailed, because this paper hypothesizes that, if supermajorities begin to present themselves before a merge, a shock to the system will make these supermajorities unstable and perhaps break them down.

At the individual level the following logit will be performed:

$$Y_2 = \beta_1 W_3 + \alpha \quad (5)$$

This regression will test the effect the merge has on individual decisions to join a supermajority. This will be extended to see the effect the merge has on path dependency of the individual decision in the following two logit procedures tests:

$$Y_2 = \beta_1 W_1 + \beta_2 W_3 + \alpha \quad (6)$$

$$Y_2 = \beta_1 W_2 + \beta_2 W_3 + \alpha \quad (7)$$

Test (6) is a combination of test (2) and (5) and test (7) is a combination of test (3) and (5). Test (6) measures the relationship of a player's decision to join a supermajority, with that player's decision the instance before while weighing in the effects the merge may have on that decision. Test (7) does the same measurement, but uses the player being in a majority the time before rather than a supermajority. The results of test (7) have slightly less economic meaning than test (6), but are used for the same reasoning as discussed with test (3).

IV. Results

Table 4. Path Dependency at the Group Level

Logit Results for Path Dependancy at the Group Level

	1
Constant	0.0606
	(0.201)
In Supermajority in Prior Instance	0.4962*
	(0.267)
Pseudo R2	0.011
N	239

Standard errors are in parentheses. *,**, and *** denote significance at the .10, .05, and .01 levels, respectively.

Table 5. Regression Results at the Individual Level

Logit Results for the Variable Effects on the Individual Choice to be in a Supermajority

	2	3	5	6	7
Constant	-0.4403***	-0.0704***	-0.0307	-0.3615***	-0.5775***
	(0.062)	(0.107)	(0.071)	(0.089)	(0.122)
In Supermajority in Prior Instance	0.6004***			0.5791***	
	(0.095)			(0.096)	
In Majority in Prior Instance		0.6501***			0.6412***
		(0.119)			(0.119)
If the Merge has Occurred			-0.2425*	-0.1188	-0.2051**
			(0.094)	(0.097)	(0.095)
Pseudo R2	0.015	0.012	0.002	0.0163	0.0138
N	1856	1856	1877	1856	1856

Standard errors are in parentheses. *,**, and *** denote significance at the .10, .05, and .01 levels, respectively.

Table 6. Effect of Shock on Supermajorities

Two-Sample test of Proportions of the effects of the Merge on Supermajorities	
	4
Supermajorities Pre-Merge	.6347 (.037)
Supermajorities Post-Merge	.5310 (.041)
Difference	.1037 (.056)
Z-Statistic	1.85
Pr(Z>z)	0.032

Standard errors are in parentheses.

V. Discussion

The first topic of discussion is if supermajorities show path dependency. The results in tests (1), (2), and (6) all indicate how supermajorities in one instance affect the likelihood of a supermajority in the following instance. All three of these results are positive, and (2) and (6) are significant at the 5% confidence level. Test (1) falls just short of the 5% confidence level (P=.063), however, this may be due to the small N, which is limited by the number of observable voting instances. Furthermore, the results in tests (3) and (7) reveal that a majority vote in the previous instance will affect the likelihood of being in a supermajority.

These results substantiate Groseclose's theory on supermajority formation. Pursuant to Groseclose's theory, the supermajorities are more prevalent then minimal winning coalitions in this data set. With supermajorities being path dependent, it shows that supermajorities are the result of the coalition formation and that those coalitions maintain their supermajorities as it is their most efficient option. If Riker's theory is correct, we would expect that when supermajorities form, they would break down because they are more expensive to maintain then the minimal winning coalitions.

The strength of supermajorities can have implications for anyone in a continuous voting game where supermajorities already exist. For example, if a supermajority is present in a board of directors for a corporation, then that supermajority could dictate every decision made, leaving little hope for the minority members to influence the direction of the corporation. The implications could be even further reaching if a supermajority forms in an institution such as Congress. If the results of this paper are indicative of how all supermajorities behave, a congressional supermajority would not only decide the outcome of a single bill, but rather the outcomes of an entire line of legislation.

By looking at the other results, we can see how the members of minority coalition may be able to disrupt the strength of path-dependent supermajorities.

The second aspect of supermajorities tested is how a shock to the voting structure affects the strength of supermajorities. Note that β_1 in test (5), β_2 in test (6) and (7), and the results of test (4), illustrate how a shock to the voter makeup does affect supermajorities. In all cases, the merge has a negative effect on the presence of supermajorities, and all the results are significant at the 5% level except the results in test (6) (p=.222).

Knowing that a shock breaks down supermajorities goes against the general theory presented by Groseclose. It can be thought, that after a shock, the voting structure goes to a more natural state, and coalitions must reform. If Groseclose theory were to hold true, then supermajorities would be expected to form in this state. However, in light of Koehler's extension to Groseclose, these results can be understood. It can be thought that the knowledge of each player's preferences become more revealed as time passes, due to observations of his interaction. After a shock, the knowledge of each player's preferences about the others is reset. With this argument, it would be assumed that before and after the merge should be identical, because each player starts with no knowledge at the first period, and then progressively learns more through each period. The difference however, is that after the merge, all of the players have more intense preferences than at the beginning of the game. In the first period of the game, it can be thought that each player has a neutral preference, and the preferences develop from that point onward. After the merge, however, each player has a developed preference, and these preferences are not unknown by half of the group. This result gives strong support for Koehler's theory, which in turn, supports Groseclose's model.

For members of minimal winning coalitions, the results knowledge of these findings can help fight the strength of supermajorities. If a member of a board of directors finds himself in a minimal winning coalition, the strength of the opposition may be too costly to attempt to persuade members of the opposition to shirk. Instead, they may want to employ strategies to shock the voting structure. This could involve lobbying stockholders to vote new members into the board. If stockholders agree with some of the minority coalition's platforms, then this could be a less expensive route to break down supermajorities. Some members of minimal winning coalitions would have possibly disregarded this strategy, claiming that there is to much uncertainty of the outcome. However, with these results, there is now empirical evidence to suggest that it would be an effective strategy.

VI. Conclusion

The results in this paper lead to many implications. Foremost, the results provide defense for the Groseclose model. By defending the Groseclose model, coalition builders can now create supermajorities with more confidence that they are creating the least expensive coalitions. Also, by defending Groseclose model with a data set that is known to contain coalition building, the findings of this paper can help detect coalition formation in other voting structures. If coalition building is unknown, or even unexpected, then the voting outcomes can be compared to the results in this paper. If the results are similar, then the other voting structure may be suspect of coalition formation.

Further research could be conducted to help detect coalition formation by performing the same type of tests of a structure that is known to not contain any coalition building. If those results vary significantly from the ones seen here, then a basis can be established to determine if a voting structure has coalition building present.

The findings in this paper can also be suggestive for members of voting structures as they form their strategies. With these findings in mind, the members now know that creating a supermajority may not only be the least costly coalition, but may also have far reaching benefits as the supermajority may persist through multiple voting instances. Furthermore, the results also suggest strategies for members who find themselves in the minority, pinned against a strong supermajority that is persisting through multiple time periods. The members of these minorities may want to allocate their CBE in methods that could bring a shock to the structure, rather than by combating the supermajority head on.

Further research may want to investigate the results of a supermajority break down when bringing new members into the structure breaks down supermajorities. In this paper, the shock contained all members that had pre-calculated preferences, which helps to support Koehler's theory of why the supermajorities break down because the knowledge of those preferences is unknown to all the players. However, in many applicable situations, bringing new members into the voting structure implies that the new members come in with neutral preferences, similar to the beginning of the Survivor. If the new members have neutral preferences, then the members of the supermajority would still have knowledge of all members' preferences. Koehler's same theory that supports the breakdown of supermajorities after a merge, would suggest there would be no breakdown if the new members begin with neutral preferences.

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VIII. Appendix

Appendix A. Model of Survivor

Voting Round		Group 1									Group 2							
1	Player	1	2	3	4	5	6	7	8		9	10	11	12	13	14	15	16
	Vote	(3)	(1)	(1)	(3)	(3)	(1)	(1)	(1)									
2											9	10	11	12	13	14	15	16
											(16)	(11)	(16)	(16)	(11)	(16)	(16)	(11)
3			2	3	4	5	6	7	8									
			(5)	(2)	(5)	(2)	(2)	(2)	(5)									
4											9	10	11	12	13	14	15	
											(15)	(15)	(15)	(15)	(10)	(10)	(10)	
5				3	4	5	6	7	8									
				(6)	(3)	(3)	(3)	(6)	(3)									
6											9	10	11	12	13	14		
											(10)	(14)	(14)	(14)	(14)	(10)		
7					4	5	6	7	8									
					(8)	(8)	(4)	(4)	(4)									
8											9	10	11	12	13			
											(13)	(9)	(13)	(13)	(9)			
											merge							
9							5	6	7	8	9	10	11	12				
							(12)	(12)	(5)	(12)	(5)	(12)	(12)	(5)				
10							5	6	7	8	9	10	11					
							(11)	(11)	(5)	(11)	(5)	(5)	(5)					
11								6	7	8	9	10	11					
								(11)	(11)	(11)	(11)	(6)	(6)					
12								6	7	8	9	10						
								(10)	(10)	(6)	(6)	(6)						
Final Four									7	8	9	10						

Appendix B. Percentage of Supermajorities