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# Computational intermediation and the evolution of computation as a commodity

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The consumer who purchases computational power ultimately purchases a reduction in the time interval between the initiation and the completion of work. This paper looks at computation as a commodity and the nascent industry of computational intermediation, and proposes a model for the market for computational power as distinct from the market for computers. Some interesting results emerge. The model implies that the demand for computation is discontinuous and that there is a lower limit to the quantity of computation consumers will demand that is independent of the price of power. The model identifies a range of computational powers that could be supplied by computational intermediaries but which will not be supplied by computer manufacturers, and suggests a model for pricing computation.

## I. INTRODUCTION

More than 30 years ago, in *The Economics of Computers*, William F. Sharpe posited models for pricing ‘rented’ computer time. At this time, computers were so expensive that firms with intermittent computational needs usually could not justify spending top dollar for a computer that would periodically sit idle. In response, entrepreneurs conceived the notion of ‘renting’ computer time.<sup>1</sup>

The advent of the low-cost personal computer made renting computer time obsolete. However, thirty years later, a similar phenomenon has arisen in the market for supercomputation. Unlike thirty years ago, two additional factors significantly alter the market dynamics: (1) the

Internet makes it possible to ‘transport’ computational power at low cost; (2) the ubiquity of personal computers combined with the tendency to buy personal computers that satisfy *peak*, not average demand for power, means that there exists a tremendous reserve of idle computational power. For example, as of January 2001, there were almost 400 million personal computers in use worldwide.<sup>2</sup> Assuming, conservatively, that the average computer has a speed of 550 Mhz, is powered-up 25% of the time, and is idle 95% of the time that it is powered-up, worldwide idle computational power is over 52 billion MHz. This is the equivalent of 7600 TF (one teraflop equals one-trillion floating point operations per second),<sup>3,4</sup> or 34 times the combined power of the world’s 500 most

<sup>1</sup> Among the entrepreneurs was a young Ross Perot who, with the founding of Electronic Data Systems, was one of the first to offer computation for rent.

<sup>2</sup> *Interactive Week*, 15 January 2001, p. 74.

<sup>3</sup> Clock-speed (i.e. cycles per second, or Hz) is frequently used as a measure for processor power in PCs. The measure is an imperfect power metric because the number of cycles required to perform a given mathematical operation varies among chip designs (this is why, for example, an 800 MHz Pentium chip is faster than an 800 MHz Celeron chip). A preferred power metric is *flops* (floating point operations per second). This metric directly measures the number of mathematical operations the chip performs per unit time. While a preferable metric to clock-speed, flops is also imperfect because (a) it is representative of effective speed only for operations that involve floating point calculations, and (b) the results obtained can vary depending on the type of mathematical tests performed. Throughout this paper, unless otherwise noted, the maximum sustained flops ratings are used (vs. peak flops rating) obtained from a Linpack inversion of a 1000 × 1000 matrix. Cf. Dongarra (2002).

<sup>4</sup> A Dell Pentium 3, 550 MHz operates at a sustained maximum power of 0.08 GF. The peak operating power – which is often quoted but rarely achieved – is 0.55 GF. Cf. The Performance Database Server, [www.netlib.org](http://www.netlib.org).

powerful supercomputers.<sup>5</sup> Note that this idle power is not *potential* power in the sense that it could be generated but isn't. Rather the power is idle in the sense that it *is* generated, but is not applied to any computational work. When a processor is powered-up but not used to its full potential, the processor does not fail to generate computational power, rather the processor generates what are called 'no-ops' (no operations) – operations that generate no output. To draw an analogy, a computer generating no-ops is like a car engine that is running at full throttle but with the transmission disengaged.

Over the past 30 years, there has been a 'decentralization' occurring in the production of computational power. Large, stand-alone supercomputers have been overtaken by cluster computers – networks of personal computers that are linked together to provide more power than traditional supercomputers at a fraction of the cost. The next step in the evolution of cluster computers has been the advent of Internet distributed computing – an industry in which a computational intermediary purchases idle processing power from individual personal computer owners (via Internet connection), bundles the power, and resells the power as high-performance computation.<sup>6</sup> This industry has emerged due to a confluence of events: (1) following Moore's Law, the computational power of personal computers is doubling (on average) every 18 months; (2) digital communications speed is doubling every 12 months;<sup>7</sup> and (3) over time, the performance gap between high-performance computers (HPC) and personal computers (PC) has closed due to dramatic improvements in integrated circuit technology.<sup>8</sup> The combination of these events creates an environment in which hardware has become so inexpensive that personal computer owners can afford to purchase levels of computational power and bandwidth that satisfy their peak demands. Because computation and bandwidth demand is extremely peaky, as PC owners purchase hardware to satisfy peak demand, the amount of idle computational power and bandwidth increases dramatically.

As Internet distributed computing becomes more ubiquitous, a 'computational grid' is evolving that provides computational power on an as-needed basis.<sup>9</sup> The computational grid heralds the transformation away from computers-as-goods to computation-as-commodity. The

purpose of this paper is to propose a model for the market for computation and to examine the implications for consumer behaviour implied by the model.

The remainder of this paper is organized as follows. The next section looks at the evolution of computation as a commodity. Section III proposes a model for deriving the demand and supply of computation. Sections IV and V draw on existing data to estimate the model parameters and apply the model to selected case studies. Section VI provides summary comments.

## II. THE EVOLUTION OF COMPUTATION AS A COMMODITY

Prior to the early 1990s, high-performance computation was conducted on 'single-machine' systems such as those produced by Cray (now, SGI), HP, IBM, Compaq, and Sun. In 1993, worldwide sales of new high-performance computers exceeded \$2.3 billion. This number had increased by an average of 12% annually to a projected \$7.5 billion in 2003.<sup>10</sup> If new sales are accumulated over time, a useful life is assumed of three to five years, and related costs of installation, infrastructure, support, maintenance, environmental controls, etc. are accounted for, one gets an estimate of worldwide expenditures on HPC of more than \$18 billion in 2003. If the low-power end of HPC (machines priced under \$250,000) are left out as being used mostly for low-computational, high-data work, annual expenditures have been projected on traditional HPC used for computation of \$12 billion in 2003.<sup>11</sup>

In an attempt to side-step the high cost of traditional HPC, computer scientists constructed one of the first *cluster computers* in 1994.<sup>12</sup> Comprised of 16 off-the-shelf personal computers connected via channel bonded Ethernet, the cluster was constructed for NASA's Earth and Space Sciences Project.<sup>13</sup> Over the past decade, cluster computers have come to provide a popular low-cost alternative to traditional HPC – the most powerful supercomputers today are cluster computers, not traditional HPC. A cluster computer can generate computational power at a purchase price of around \$4000 per gigaflop (GF). For example, a cluster comprised of off-the-shelf 1.8 GHz Pentium 4 computers generates approximately 0.24 GF per node<sup>14</sup>

<sup>5</sup> Cf. [www.top500.org](http://www.top500.org), June 2002.

<sup>6</sup> Within the industry, the legacy terms 'super computation (computer)' have been supplanted by 'high-performance computation (computer),' or 'HPC.'

<sup>7</sup> Roberts (2000).

<sup>8</sup> Baskett and Hennessey (1993).

<sup>9</sup> Cf. Hawick *et al.* (1998).

<sup>10</sup> International Data Corporation, Report #W20058, September 1999. Chris Willard, Vice President of Research for IDC, indicates that the sales numbers in W20058 understates true sales for 1993 and 1994.

<sup>11</sup> Davies (2001b).

<sup>12</sup> The legacy term for 'cluster computer' is 'Beowulf cluster,' taken from the name of one of the first cluster computers.

<sup>13</sup> See [www.beowulf.org](http://www.beowulf.org) for a synopsis of the early history of cluster computers.

<sup>14</sup> Wasson (2001). [www.tech-report.com](http://www.tech-report.com)

and costs around \$928 per node<sup>15</sup> for a purchase-price-performance ratio of approximately \$3900 per GF.<sup>16</sup> By comparison, traditional HPC generates power for around \$10 000 to \$25 000 purchase-price-per-GF.<sup>17,18</sup>

While cluster computers offer a lower-cost alternative to traditional HPC, cluster computers require the same infrastructure and amortization expenses as traditional HPC. By linking together computers via the Internet, a virtual cluster computer (or 'computer grid') can be created at, effectively, zero infrastructure expense. More specifically, the infrastructure expense is borne by the individuals who own the computers. Since Internet computing taps idle computer power and uses bandwidth opportunistically, the marginal infrastructure expense borne by the individual computer owner is zero. The proof-of-concept for the Internet distributed computation industry was the non-profit venture SETI@Home.<sup>19</sup> This project was born when astronomers, attempting to sift through volumes of data from the Aracibo Radio Telescope, were unable to acquire computers powerful enough to analyse their data. Researchers devised a 'screen saver' that PC owners would download and install on their machines. When a participant's computer was idle and the user was on-line, the software would reach out across the Internet, retrieve a small chunk of the radio-telescope data, analyse the data, and return the results. Principally through word-of-mouth, the researchers received offers from thousands of individuals who wanted to donate their computers' idle times. By the end of the first 18 months of operations, more than

450,000 computers were running SETI@Home's screen saver. SETI@Home currently claims over 3.4 million participants contributing, collectively, almost 30 teraflops (TF) of computing power at a total cost of \$500,000—or \$17 per GF. By comparison, ASCI White, until recently the world's most powerful supercomputer, generates 7.2 TF of power at a purchase price of \$110 million—or over \$15,000 per GF.<sup>20,21</sup>

Using SETI@Home as proof-of-concept, over the past four years, commercial and non-profit ventures such as IBM's Grid Computing Initiative, the European Grid Forum, Intel's Peer-to-Peer Computing Initiative, Sun's One Grid, Parabon Computation, Data Synapse, United Devices, and Entropia have evolved to capitalize on the intermediation of idle computation.<sup>22</sup> Early successes include projects involving gene sequence comparisons, exhaustive statistical analyses, and virtual drug screening.<sup>23</sup>

To date, only Parabon Computation and United Devices offer software development kits (SDK) for Internet computing making these firms the only 'up and running' commercial computational intermediaries.<sup>24</sup> With an SDK, a programmer writes a program and launches the program from his computer. The program is received by the intermediary's server and is distributed to thousands of PCs around the world. As the individual PCs complete their work, the results are uploaded to the intermediary's server where the programmer can retrieve them. Interestingly, these firms charge for computational power on a

<sup>15</sup> As of January 2002, a headless, 1.8 GHz, Dell Pentium 4 with 256 SDRAM retailed for \$928. The Linpack rating (matrix size = 1000) for this processor is approximately 0.24 GF.

<sup>16</sup> Examples include (numbers are shown in present value, annual cost terms, adjusting for inflation and Moore's Law): The Danforth Cluster ([www.danforthcenter.org](http://www.danforthcenter.org)) at \$5,700 per GF, Loki ([loki-www.lanl.gov](http://loki-www.lanl.gov)) at \$4,500 per GF, SWARM ([www.cs.orst.edu](http://www.cs.orst.edu)) at \$5,100 per GF, and Brahma ([www.phy.duke.edu](http://www.phy.duke.edu)) at \$4,100 per GF.

<sup>17</sup> Joseph *et al.* (2000c).

<sup>18</sup> Price-performance numbers are usually quoted in terms of purchase price. The actual *annual* cost of a cluster computer (including amortized purchase price, infrastructure, maintenance, and support) is around 125% of the purchase price of the nodes (cf. Davies, 2001a).

<sup>19</sup> See [www.setiathome.ssl.berkeley.edu](http://www.setiathome.ssl.berkeley.edu).

<sup>20</sup> See [www.top500.org](http://www.top500.org). Note that several power figures are listed for the Top 500 computers. Among them are the maximum sustained power and the peak power. In most public relations literature, peak power is quoted. Peak power has been described as 'the lowest speed the machine is guaranteed not to achieve.' Throughout this paper, the maximum sustained power is used. Note that, on average, peak power is 1.5 times sustained maximum power.

<sup>21</sup> ASCI White's purchase-price-performance ratio is on a par with that of traditional HPC because ASCI White is, strictly speaking, a hybrid machine. It is a cluster computer whose nodes are not off-the-shelf PCs, but are themselves low-power traditional HPC. Thus, ASCI White's price-performance ratio is on a par with traditional HPC. What ASCI White gains through clustering is a power level unattainable by traditional HPC. Cf. <http://www-1.ibm.com/servers/eserver/pseries/hardware/largescale/supercomputers/asciwhite/>

<sup>22</sup> See IBM ([www-1.ibm.com/servers/events/grid.html](http://www-1.ibm.com/servers/events/grid.html)), EGrid ([www.egrid.org](http://www.egrid.org)), Intel ([www.intel.com/ids/p2p](http://www.intel.com/ids/p2p)), Sun ([www.sun.com/software/gridware](http://www.sun.com/software/gridware)), Parabon Computation ([www.parabon.com](http://www.parabon.com)), DataSynapse ([www.datasynapse.com](http://www.datasynapse.com)), United Devices ([www.uniteddevices.com](http://www.uniteddevices.com)), and Entropia ([www.entropia.com](http://www.entropia.com)).

<sup>23</sup> See, for example, [www.computeagainstcancer.org](http://www.computeagainstcancer.org), and [www.fightaidsathome.org](http://www.fightaidsathome.org).

<sup>24</sup> Sun and Datasynapse offer SDK's for 'enterprise computing grids.' An enterprise computing grid is like an Internet computing grid with the exception that all of the computers on the grid are 'in-house.' Thus, an enterprise computing grid is a middle-step between a cluster computer and an Internet computing grid: the consumer of the computation owns the computers, but the computation generated is idle capacity. In the case of an enterprise computing grid, the consumer does incur infrastructure and amortization costs but, because the grid harnesses idle capacity (and ignoring the cost of the software that administrates the grid), the marginal increase in infrastructure and amortization from establishing the grid is zero.

per-hour-used basis, making both the delivery and pricing of computation analogous to that of electricity.<sup>25</sup>

### III. MODELLING DEMAND AND SUPPLY OF COMPUTATION

The model proposed below is generic on the demand side and cluster-specific on the supply side. That is, the derivation of demand is applicable to any consumer of computation, regardless of the means by which the computation is generated (i.e. traditional HPC, cluster computer, or computational intermediation). Because cluster computing is a lower cost alternative to traditional HPC, supply is initially derived assuming that the computation is produced via a cluster.

#### *Computation supplied via cluster computer*

Suppose a consumer has a single task, the completion of which requires  $W$  gigaflop-years (GFY) of computational work, where one GFY is the amount of computation performed by a 1 GF computer running continuously for one year.<sup>26</sup> When completed, the work will have a value of  $\Omega$  dollars. If computational power  $P$  GF is applied to the work, then the work will complete in  $W/P$  years. At an annual subjective discount rate  $r$ , the present value of the completed work is (where  $PV_0$  means ‘present value, in today’s dollars, of work initiated today’):

$$PV_0(\text{completed work}) = \Omega e^{-rW/P} \quad (1)$$

Suppose that the consumer purchases a cluster today. The cluster will be comprised of a set of computers (called ‘nodes’), each of which has an expected useful life of  $n$  years. Let  $k_t$  be the present value (on a per-GF basis), for a node purchased  $t$  years in the future, of the purchase, installation, maintenance, and related infrastructure costs associated with the node and incurred over the node’s life. According to Moore’s Law, the price–power ratio will grow annually at rate  $\delta$  (hereafter, ‘Moore’s rate’).<sup>27</sup> Applying Moore’s rate, the price–power ratio for a node purchased  $t$  years in the future is:

$$k_t = k_0 e^{\delta t} \quad (2)$$

As each node reaches the end of its useful life, it is replaced by new node. If the life of the cluster is the length of time

required to complete the work  $W$ , then all the nodes comprising the cluster will be replaced every  $n$  years from today to  $W/P$  years in the future. For the sake of simplicity, let us assume that the cluster is dismantled at the completion of the work (i.e.  $W/P$  years in the future) for zero cost and at zero salvage value. The present value of the price–power ratio for the cluster purchased today and dismantled  $W/P$  years from now is:

$$\begin{aligned} PV_0(\text{price–power ratio}) &= k_0 + k_0 e^{(\delta-r)n} + k_0 e^{2(\delta-r)n} + \dots + k_0 e^{(W/P)(\delta-r)n} \\ &= k_0 \sum_{i=0}^{W/P} \left( e^{(\delta-r)n} \right)^i = \frac{k_0 \left( e^{nr} - e^{n((\delta-r)W/P + \delta)} \right)}{e^{nr} - e^{n\delta}} \end{aligned} \quad (3)$$

Multiplying Expression 3 by  $P$ , one gets the cost (in present value terms) of a cluster of power  $P$  GF purchased today:

$$PV_0(\text{cost of power } P \text{ cluster}) = \frac{Pk_0 \left( e^{nr} - e^{n((\delta-r)W/P + \delta)} \right)}{e^{nr} - e^{n\delta}} \quad (4)$$

Taking the first derivatives of Expressions 1 and 4, one has the marginal benefit and marginal cost of power (in present value terms):

$$\text{Marginal benefit of additional power} = \frac{\Omega r W}{P^2 e^{rW/P}} \quad (5)$$

$$\begin{aligned} \text{Marginal cost of additional power} \\ = k_0 \frac{e^{nr} + ((n(\delta-r)W/P) - 1)e^{n((\delta-r)W/P + \delta)}}{e^{nr} - e^{n\delta}} \end{aligned} \quad (6)$$

Taking the derivative of Equation 5 with respect to  $P$ , it is found that the marginal benefit of computation *increases* with power for values of  $P$  less than  $rW/2$ . Taking the derivative of Equation 6 with respect to  $P$ , it is found that the marginal cost of additional power falls asymptotically to  $k_0$ , first at an increasing, then at a decreasing rate.<sup>28</sup> Equating Expressions 5 and 6 does not yield a closed-form solution for  $P$ . Let us use an approximation to Expression 6 by assuming that the cluster exists for an infinite amount of time. One has:

$$\begin{aligned} PV_0(\text{cost of power } P \text{ cluster}) \\ = P \left( k_0 + k_0 e^{(\delta-r)n} + k_0 e^{2(\delta-r)n} + \dots \right) = \frac{Pk_0}{1 - e^{(\delta-r)n}} \end{aligned} \quad (7)$$

Because future costs of the cluster are discounted both by the present value calculation and by the decrease in

<sup>25</sup> For example, from a laptop computer connected to the Internet via a phone line, a statistical program has been run on Parabon Computation’s platform that was distributed to 5000 PCs. Thirty-six hours later, the entire job had completed and the results were downloaded from Parabon’s server. Run on a single computer, the job would have required ten years to complete.

<sup>26</sup> Because GF are measured on a per-unit-time basis (i.e. floating-point operations per second), multiplying by years cancels the time dimension. Thus, 1 GFY is approximately  $3.15 \times 10^{16}$  floating-point operations.

<sup>27</sup> Gordon Moore, co-founder of Intel, predicted in 1965 that the number of transistors per square inch (called *data density*) on integrated circuits would double every year. The pace of increase in data density subsequently slowed to a doubling every 18 months. Note that Moore’s Law implies  $\delta < 0$ .

<sup>28</sup> The inflection point occurs at  $P = nW(r - \ln(\delta))/3$ .



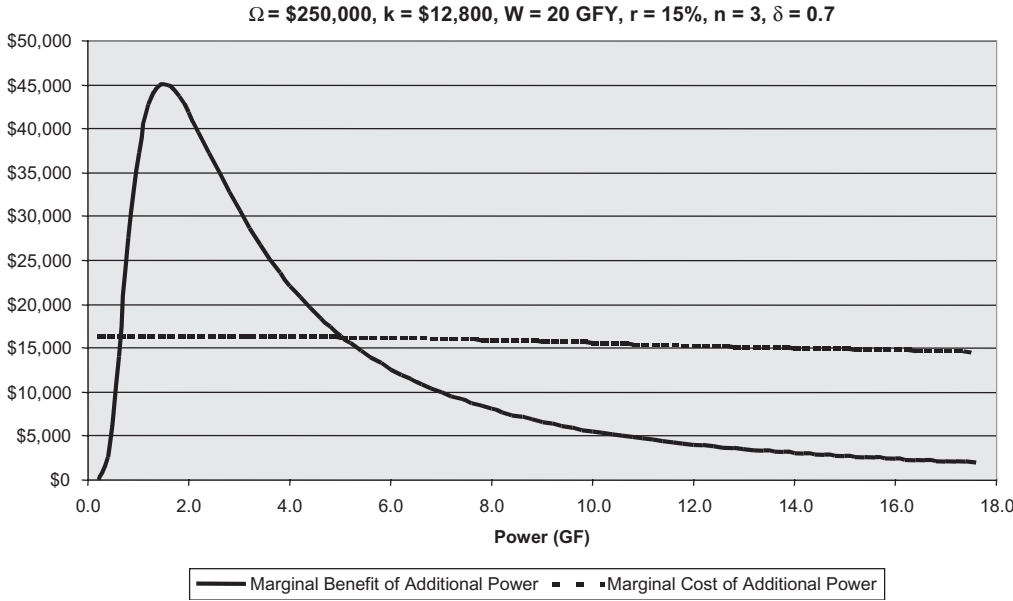


Fig. 2. Marginal benefit and marginal cost of computational power

W-function in Equation 8 is negative. Values of  $\omega_0(x)$  for  $x < -1/e$  correspond to marginal cost exceeding marginal benefit for all non-negative values of  $P$ . For example, at  $\omega_0(-1/e)$ , Equation 8 becomes  $P^* = -rW/[(2)(-1)] = rW/2$ , which is the power level that maximizes marginal benefit.

Figure 2 shows the marginal benefit and marginal cost of additional computational power. The marginal cost curve shown is from Equation 6. In the figure, marginal benefit and marginal cost intersect at  $P = 5.031$ . Using the approximation for marginal cost derived from Equation 7, the closed form solution for  $P^*$  in Equation 8 yields  $P = 5.018$ .

Two interesting results emerge from this analysis. First, because marginal benefit has an upward sloping section, the demand for computation is discontinuous at power level  $rW/2$ ; consumers will either purchase zero power or some quantity of power greater than  $rW/2$ . The reason is that power is actually a ‘shadow product.’ The product the consumer is really purchasing is a reduction in the time interval between initiation and completion of the computational work. If one thinks of time as the product the consumer is actually purchasing, then the present value of the benefit of time,  $T$ , is  $\Omega e^{-rT}$ , and the marginal benefit of time is  $-\Omega r e^{-rT}$  (the marginal benefit is negative because a decrease in time increases the present value of the completed work). Thus, the marginal benefit of additional reductions in time falls as the size of the time reduction increases. Because time reduction is consumed indirectly in the form of power consumption, and because the change in time given a change in power varies differently than does the change in benefit given a change in time, we get the odd result of increasing marginal benefit over some ranges of power.

We can identify the source of the upward sloping marginal benefit by decomposing marginal benefit into its constituent functions (where ‘present value’ is understood). One has:

$$\frac{\partial \text{Benefit}}{\partial \text{Power}} = \left( \frac{\partial \text{Benefit}}{\partial \text{Time}} \right) \left( \frac{\partial \text{Time}}{\partial \text{Power}} \right) \quad (13)$$

where the three terms in Equation 13 are, respectively, the marginal benefit of power, the marginal benefit of time, and the marginal time of power (i.e. the change in the time required for completion given an instantaneous change in power). The marginal benefit of time and its derivative are given by:

$$\begin{aligned} \frac{\partial \text{Benefit}}{\partial \text{Time}} &= \text{Marginal Benefit of Time} \\ &= -\Omega r e^{-rT} < 0 \quad \forall \Omega, r, T > 0 \end{aligned} \quad (14)$$

$$\frac{\partial \text{Marginal Benefit of Time}}{\partial \text{Time}} = \Omega r^2 e^{-rT} > 0 \quad \forall \Omega, r, T > 0$$

The marginal time of power and its derivative are given by:

$$\begin{aligned} \frac{\partial \text{Time}}{\partial \text{Power}} &= \text{Marginal Time of Power} \\ &= -\frac{W}{P^2} < 0 \quad \forall W, P > 0 \end{aligned} \quad (15)$$

$$\frac{\partial \text{Marginal Time of Power}}{\partial \text{Power}} = \frac{2W}{P^3} > 0 \quad \forall W, P > 0$$

Thus, as power increases, time decreases at a decreasing rate, but as time decreases, benefit increases at an

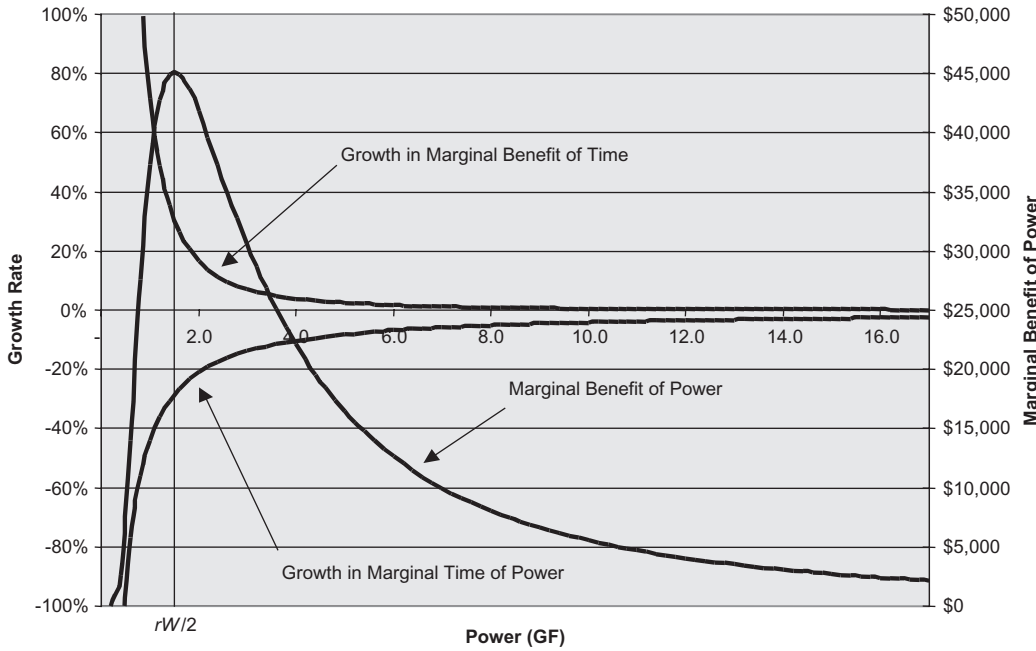


Fig. 3. Growth rates in components of the demand for power

increasing rate. Taking the derivative of Equation 13 with respect to power, one has:

$$\begin{aligned} & \frac{\partial \text{Marginal Benefit of Power}}{\partial \text{Power}} \\ &= (\text{Marginal Benefit of Time}) \\ & \times \left( \frac{\partial \text{Marginal Time of Power}}{\partial \text{Power}} \right) \\ &+ (\text{Marginal Time of Power}) \\ & \times \left( \frac{\partial \text{Marginal Benefit of Time}}{\partial \text{Power}} \right) \end{aligned} \quad (16)$$

Setting Equation 16 equal to zero and simplifying, one has:

$$\frac{\partial \text{Marginal Benefit of Time}}{\text{Marginal Benefit of Time}} = - \frac{\partial \text{Marginal Time of Power}}{\text{Marginal Time of Power}} \quad (17)$$

The left-and right-hand sides of Equation 17 are (approximately) the percentage change in the marginal benefit of time, and the percentage change in the marginal time of power, respectively. As power increases, the marginal benefit of time declines asymptotically to  $-r\Omega$ , while the marginal time of power increases asymptotically to zero. Because the marginal benefit of power is the product of

these, marginal benefit of power declines asymptotically to zero. But at low power levels (i.e.  $P < rW/2$ ), the growth rate in the marginal benefit of time exceeds the rate of decline in the marginal time of power (Fig. 3).<sup>33</sup> So, overall, an increase in power results in an increase in the marginal benefit of power. As power increases, eventually the marginal benefit of time starts to grow at a slower rate, proportionally, than the marginal time of power declines. From this point forward, further increases in power impact the growth rate of time more than the reduction in time impacts the growth rate of benefit.

The second interesting result is that (for reasonable parameter values) the marginal cost of computation is downward sloping. Taking the derivative of Expression 6 with respect to  $P$ , setting the derivative equal to zero, and solving for  $\delta$ , gives one the result that marginal cost is downward sloping for all positive  $P$  when  $r > \delta$ . Two forces impact the change in the cost of power over time: (1) because of Moore’s Law, nodes purchased in the future cost less (on a per-unit-power basis) than nodes purchased in the past; (2) because of present value discounting, dollars spent in the future are worth less than dollars spent in the present. As long as Moore’s rate works to decrease the cost of power (i.e.  $\delta < 0$ ), and interest rates are positive, one is guaranteed that  $r > \delta$ , and so the marginal cost of power declines as power increases.<sup>34</sup>

<sup>33</sup> Figure 3 is constructed using the parameter values shown in Fig. 2.

<sup>34</sup> One might hypothesize the case in which Moore’s Law is exhausted and so the cost of power rises with inflation. Because the discount rate can be taken to be the nominal interest rate, and assuming positive real interest, we would expect that the condition  $r > \delta$  would continue to hold.

Suppose that one observes a consumer purchase a cluster with power  $P^*$ . Assuming the consumer has purchased the optimal quantity of power, Equation 8 can be used to infer the value of the completed work to the consumer. Solving for  $\Omega$  (and noting that  $\omega(x)e^{\omega(x)} = x$ ), one has:

$$\begin{aligned} & \text{Estimated implied value of completed work} \\ & = \Omega = \frac{k_0 e^{(rW/P^*)} P^{*2}}{rW(1 - e^{(\delta-r)n})} \end{aligned} \quad (18)$$

The qualifier ‘estimated’ is used because Expression 18 is an estimate of optimal power given the simplifying assumption in Equation 7 that the cluster has infinite life. Converting to present value terms and subtracting the (estimated) present value of the cost of the cluster in Equation 7 one has the (estimated) implied present value of the completed work net of the cost of the cluster:

$$\begin{aligned} & \text{Estimated implied net present value of completed work} \\ & = \frac{k_0 P^{*2}}{rW(1 - e^{(\delta-r)n})} - \frac{k_0 P^*}{1 - e^{(\delta-r)n}} = \frac{k_0 P^*(P^* - rW)}{rW(1 - e^{(\delta-r)n})} \end{aligned} \quad (19)$$

Substituting the optimal power from Equation 8 into Equation 19, setting (19) equal to zero, and solving for  $\Omega$ , yields the necessary condition for purchasing any positive quantity of power.

Minimum value of completed work required for consumer to purchase any positive quantity of power:

$$\Omega > \frac{k_0 r W e}{1 - e^{(\delta-r)n}} \quad (20)$$

(where, in the numerator,  $e = 2.71828\dots$ ). Computation will not be applied to any project whose future value does not satisfy condition 20. Evidence collected by Nordhaus (2002) suggests that, since 1950, the price–power ratio  $k_0$ , has fallen by approximately nine orders of magnitude in real terms.<sup>35</sup> This evidence suggests that  $k_0$  is declining by 35% annually, which means that, over time, not only are existing consumers of computation purchasing more (as computation becomes less expensive), but also new consumers, who previously found computation prohibitively expensive given the value of completed work, are entering the market. If, as is reasonable,  $\Omega$  were distributed across consumers such that there is more lower-valued work than higher-valued work, we would expect the number of new consumers to rise exponentially with the fall in  $k_0$ . Examples of this are found in the growing list of applications for computational discovery. Years ago, the use of high-performance computation was restricted to

applications that attracted the greatest research dollars such as nuclear testing, weather forecasting, cryptography, and fluid dynamics. As the cost of computation has fallen, new ‘lesser-profit’ applications have evolved such as: data mining, protein folding, traffic flow modelling, inventory management, artificial speech, and speech recognition.<sup>36</sup> An interesting side effect has also emerged. As the cost of computation declines, it is becoming more cost effective to conduct ‘dumb’ searches that require less programming skill and more computing power than to search smartly, which requires more programming skill though less computing power. That is, computer users are finding it profitable to replace programmers with raw computational power.<sup>37</sup>

Substituting the limiting condition in Equation 20 into Equation 18, one has:

Minimum computation a consumer will purchase:

$$P^* > rW \quad (21)$$

That is, there exists a discontinuity such that the consumer will never consume a quantity of power less than  $rW$ , and, by extension, consumers will either wait less than  $1/r$  years for work to complete or will wait an infinite time (i.e. the work will never be completed because no power will be purchased). The discontinuity is shown in Fig. 4 where the range of powers to the left of  $rW$  yields negative net present values.

#### *Computation supplied via computational intermediary*

Suppose the computational intermediary charges for power on a per-unit-time basis, where the cost per year for  $P$  GF of power is:

$$\text{Annual cost of } P \text{ GF of power} = aP^b \quad \forall a > 0, b > 1 \quad (22)$$

Applying power  $P$ , the work requires  $W/P$  years to complete. The total cost of the computation is then:

Cost of completing  $W$  GFY using  $P$

$$\text{GF} = aP^b \left( \frac{W}{P} \right) = aP^{b-1} W \quad (23)$$

Notice that, if  $b < 1$ , the consumer will purchase an infinite amount of power. This is because an increase in power causes the time required for completion to fall proportionally more than the price per unit time rises. The result is that the cost of completing the work falls as power rises. Thus, the consumer can reduce the cost of completing the work to zero by employing infinite power. Similarly, if  $b = 1$ , the consumer’s cost is constant regardless of the

<sup>35</sup> Nordhaus (2002).

<sup>36</sup> Cf. Brown (2000), Ezzell (2000), Howard (2000), Langreth and Adams (2000), Meieran (2000) and Thayer (2000).

<sup>37</sup> Cf. Joseph *et al.* (2000d) and Pescovitz (2000).

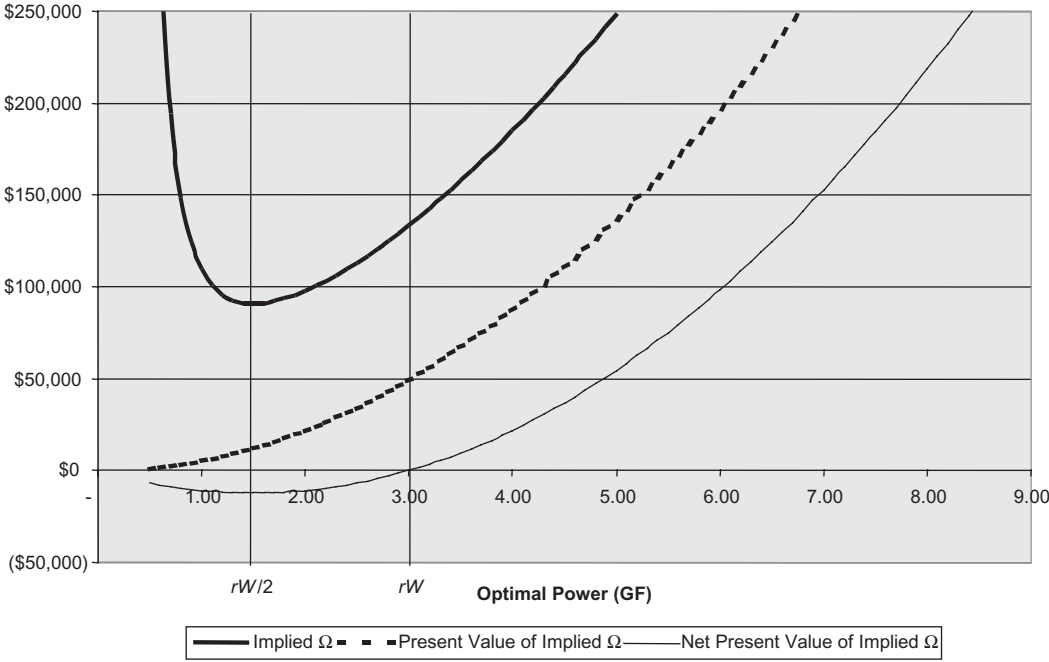


Fig. 4. Implied future value, present value, and net present value of work

power. Because increasing power increases the present value of the completed work asymptotically to  $\Omega$ , while cost remains constant, the consumer will either purchase no power (if  $\Omega < aP^{b-1}W$ ) or an infinite quantity of power (if  $\Omega > aP^{b-1}W$ ).

Differentiating Equation 23 with respect to  $P$ , one has the marginal cost to the consumer of additional power:

$$\text{Marginal Cost of Additional Power} = a(b-1)P^{b-2}W \quad (24)$$

Finally, setting Equations 5 and 24 equal, one has optimal power under the Internet computing pricing scheme:<sup>38</sup>

$$P^* = \left( \frac{\Omega r}{a(b-1)} \right)^{1/b} e^{\omega_0(-rW/b)[\Omega r/(a(b-1))]^{-1/b}} \quad (25)$$

Assuming the consumer has purchased the optimal quantity of power, we can solve Equation 25 for  $\Omega$  to find the implied value of the completed work.

$$\text{Implied value of completed work} = \frac{a(b-1)}{r} (P^*)^b e^{rW/P^*} \quad (26)$$

Taking the present value of Equation 26 and subtracting 23, one has the implied net present value of the completed work.

Implied net present value of completed work

$$= \frac{a(b-1)}{r} (P^*)^b - aW(P^*)^{b-1} \quad (27)$$

Restricting the implied net present value to be positive and substituting for the optimal quantity of power implies the minimum value requirement for the completed work.<sup>39</sup>

Minimum value of completed work required for consumer to purchase any positive quantity of power:

$$\Omega > \frac{a(b-1)e^{b-1}}{r} \left( \frac{rW}{b-1} \right)^b \quad (28)$$

This also implies that the minimum power the consumer will purchase is given by

Minimum computation a consumer will purchase:

$$P^* > \left( \frac{rW}{b-1} \right) e^{((b-1)/b) + \omega_0(((1-b)/b)e^{(1-b)/b})} \quad (29)$$

Comparing Equations 29 and 21, it is seen that the minimum computation a consumer might purchase from a computational intermediary is less than the minimum computation a consumer might purchase via a cluster computer when  $b > 2$ . This implies that the range of powers from 0 to  $rW$  that the consumer would not purchase via a cluster computer, the consumer might purchase via a computational intermediary.

<sup>38</sup> The solution in Equation 25 reduces to the solution in Equation 8 for  $b=2$ , and  $a = k_0[W(1 - e^{(\delta-r)n})]^{-1}$ .

<sup>39</sup> In the limit, as  $b \rightarrow 1$ , Equation 28 approaches  $\Omega > aW$ .

#### IV. ESTIMATING MODEL PARAMETERS

The model parameters to be estimated are  $W$ ,  $r$ ,  $\delta$ ,  $n$ , and  $k_0$ . Because work  $W$  is consumer specific and will vary significantly, it is left unestimated. The discount rate  $r$  can be estimated as the cost of capital for the consumer. While this number will also be consumer specific, it can be taken to be approximately 15%. Joseph *et al.* (2000) estimate the mean life of traditional high-performance computers to be 5.4 years (across all market segments), with a standard deviation of 2.1 years. The maximum life is 11 years; the minimum is 2. Because clusters are comprised of off-the-shelf PCs, let us assume (at the lower end of the life expectancy range) a useful node life of 3 years.<sup>40</sup> Moore's Law indicates that processor power doubles every 18 months. Nordhaus (2002) estimates that the price–power ratio for microprocessors has fallen by nine orders of magnitude, in real terms, since 1950.<sup>41</sup> Adjusting Nordhaus' estimate for inflation, one gets an estimate for Moore's rate  $\delta$ , of  $-0.35$ . The present value of the cost per GF for a cluster  $k_0$ , includes the purchase of the nodes, miscellaneous hardware (including racks, cables, routers, LAN cards, front-end nodes, KVM switches, etc.), software (including operating systems and software to distribute and manage jobs across the nodes), service and support contracts, labour, climate control, fire control, power control and filters, power and data back-up systems, space, renovation (raised floors, drop ceilings, sound insulation, rack installation), security systems, and power.

*Space:* A single node requires 0.8 square feet of space (including access).<sup>42</sup> The space must be configured for fire control, environmental control, power control, noise control, and security. The space must also be able to support the weight of the nodes (on average, 60 pounds per node including racks and accessories).<sup>43</sup> Properly configured space can cost as much as \$17 to \$25 per square foot annually.<sup>44</sup>

*Installation and configuration:* As an example, a team of skilled technicians required six weeks to install the 550-

node Danforth cluster. This cluster was assembled from off-the-shelf components.<sup>45</sup>

*Power:* One single-processor 450 MHz Pentium III computer uses \$20 worth of electricity annually.<sup>46</sup> Newer 2 GHz dual-processor nodes can consume more than three times this much power.

*Climate control:* A dedicated cluster produces a significant amount of heat. Climate control costs are approximately \$6 per single-processor node annually.<sup>47</sup>

*Miscellaneous hardware:* The cost of additional hardware can exceed 15% of the purchase price of the nodes.

*Software:* Software that distributes tasks across the cluster, monitors the job run, and collects results can cost from \$500 to \$1400 per processor.<sup>48</sup>

*Service contracts:* Software service contracts typically cost 20% of the purchase price of the software annually. Hardware service contracts can average 15% of the purchase price of the hardware annually.

*Labour:* The cluster will require at least one full-time IT specialist. This annual labour cost can be as high as \$250 to \$300 per node.<sup>49</sup>

Assuming a useful life of three years, a 15% cost of capital, and accounting for software support and maintenance, the figures above imply that the annual cost of a cluster can be from 100% to 150% of the purchase price of the computers.<sup>50</sup> For example, a cluster whose nodes (alone) are priced at \$100 000 will cost the owner \$100 000 to \$150 000 annually over three years. Based on the purchase price of a 'headless' 1.8 GHz Dell Pentium 4,<sup>51</sup> the purchase-price–performance ratio is approximately \$3900 per GF. Assuming a purchase price of \$3900 per GF, a total annual cost of 125% of the purchase price, a useful life of three years, and discounting at an estimated cost of capital of 15%, the total cost per GF of a cluster computer over its useful life is:

$$PV_0(\text{Cost per GF for a cluster computer}) = k_0 = \$12\,800 \quad (30)$$

<sup>40</sup> Cf. Joseph *et al.* (2000a).

<sup>41</sup> Cf. Nordhaus (2002). Note that inflation does not matter because it impacts both  $k$  and  $\Omega$ .

<sup>42</sup> This assumes that the nodes are stacked in a rack configuration. This configuration occupies the minimum possible floor space.

<sup>43</sup> Spector (2000) and 'Case in point: power plant', *Genome Technology*, May 2001.

<sup>44</sup> This is based on an upfront cost of \$100 to \$150 per square foot amortized at a 15% cost of capital over 15 years.

<sup>45</sup> *Genome Technology*, May 2001, p. 22.

<sup>46</sup> In the steady-state, a 450 MHz Pentium III draws 0.3 amps. This power consumption excludes the monitor because, in a cluster computer, the nodes do not require monitors.

<sup>47</sup> For example, Genetic Programming, Inc. ([www.genetic-programming.com](http://www.genetic-programming.com)) has a 1000 node (450 MHz) cluster that requires two 24-ton air conditioners. At 7 cents per kWh the cost of air conditioning is \$123 per ton annually ([www.eren.doe.gov](http://www.eren.doe.gov)).

<sup>48</sup> For example, software for the 1,040 processor Danforth Cluster costs \$800 000 per year (*Genome Technology*, May 2001, p. 22).

<sup>49</sup> Assuming one full-time IT professional per 300 nodes at an annual salary of \$70 000 plus 20% employer burden.

<sup>50</sup> Spector (2000); 'Case in point: power plant,' *Genome Technology*, May 2001; 'High-Performance Computing Industry Changes,' International Data Corporation, Telebriefing, 11 May 2001.

<sup>51</sup> As of January 2002, a headless, 1.8 GHz Dell Pentium 4 retailed for \$928 and generates 0.24 GF.

Table 2. Case studies

|                                     | Power (GF) | PV cost over life <sup>52</sup> | PV cost per GF         | Estimated life-time work (GFY) <sup>53</sup> |
|-------------------------------------|------------|---------------------------------|------------------------|--|
| ASCI white <sup>54</sup>            | 7000       | \$95.8 million                  | \$13 700               | 17 850                                       |
| Danforth cluster <sup>55</sup>      | 335        | \$3.7 million                   | \$11 000               | 854  |
| Genomics firm #1 <sup>56</sup>      | 979        | \$11.6 million                  | \$11 900               | 2 496  |
| Financial institution <sup>56</sup> | 360        | \$5.9 million                   | \$16 300               | 918  |
| Genomics firm #2 <sup>56</sup>      | 150        | \$3.7 million                   | \$24 500 <sup>57</sup> | 383  |

Table 3. Revealed value of computation to the consumer

|                       | Implied value | Minimum value |
|-----------------------|---------------|---------------|
| ASCI white            | \$440 million | \$119 million |
| Danforth cluster      | \$21 million  | \$6 million   |
| Genomics firm #1      | \$61 million  | \$17 million  |
| Financial institution | \$23 million  | \$6 million   |
| Genomics firm #2      | \$9 million   | \$3 million   |

## V. CASE STUDIES

Let us turn now to five cases: ASCI White, the Danforth cluster, two leading genomics companies, and a Global 200 financial institution. Details on these systems are given in Table 2.

To estimate the lifetime work of the clusters accounting for downtime due to maintenance, it is assumed that the clusters are available 85% of the time. Assuming that the powers of these clusters are optimal, one can use Equation 18 to estimate the implied value of the work to be performed on these clusters, and Equation 20 to estimate the minimum value required for the consumers to purchase any power at all.

## I. CONCLUSIONS

More than 30 years ago, in *The Economics of Computers*, William F. Sharpe posited models for pricing ‘rented’ computer time. Since that time, the combination of (1) a growing idle capacity of PCs, (2) a growing appetite for computational power for scientific research and computational discovery, (3) a growing interconnectedness of individual computers via the Internet, and (4) a growing

capacity for transmitting data over the Internet, has resulted in conditions that are favourable for the evolution of computational intermediaries. Following those early firms that rented computer time, computational intermediaries transform the product market for computers into the commodity market for computation. This paper offers a model for analysing the market for computation supplied via a computational intermediary and via a cluster computer. The model suggests that the demand for computation is discontinuous, that the supply of computation provided via a cluster computer can be downward sloping, and that there is some minimum quantity of power that consumers will purchase that is independent of the price of computation. These results imply that:

1. Computation is a shadow product for time such that the demand for computational power is, ultimately, driven by the demand for a reduction in time between the present and the completion of work.
2. The marginal benefit of computational power is the product of the marginal benefit of time and the marginal time of power. When an increase in power causes the marginal benefit of time to rise by more than the marginal time of power falls, the result is an overall increase in the marginal benefit of power. The marginal benefit of power is upward sloping over the range  $P = (0, rW/2)$ , and so demand is discontinuous at  $P = rW/2$ .

For computation supplied via a cluster computer:

3. The consumer will purchase either zero power or more than  $rW$  gigaflops of power. Noteworthy is that the lower limit  $rW$  is not a function of Moore’s Law – that is, the change in the cost-per-unit-power of computers has no impact on the minimum power demanded. Further, the larger the work, the greater

<sup>52</sup>This is 125% of the reported purchase price of the nodes. In the case of ASCI White, the reported cost of \$110 million is taken as including installation, infrastructure, support, and maintenance. Present value calculations assume a useful life of three years.

<sup>53</sup>This assumes that the machines are operated 85% of the time. The partial utilization reflects down time for periodic maintenance and upgrades. Cf. Hawick *et al.* (1998).

<sup>54</sup>See [www.llnl.gov/asci/news/white\\_news.html](http://www.llnl.gov/asci/news/white_news.html).

<sup>55</sup>See [www.danforthcenter.org](http://www.danforthcenter.org) for details on the cluster.

<sup>56</sup>These firms have asked not to be identified. The financial institution is a Global 200 corporation. Genomics Firm #1 is one of the leading genomics companies in the US. Genomics Firm #2 is a medium-sized firm with sales in 2000 of \$150 million.

<sup>57</sup>This company’s cluster computer is comprised of Alpha computers. Alphas have a significantly greater price-performance ratio.

the lower limit on the power the consumer will purchase.

4. By extension, the consumer will either wait an infinite time for the work to complete (i.e. the consumer will never initiate the work), or the consumer will wait less than  $1/r$  years.
5. While Moore's Law does not impact the minimum power the consumer will purchase, Moore's Law does impact the types of work to which computational power is applied. The consumer will not apply power to work worth less than  $k_0 r W e / (1 - e^{(\delta-r)n})$  in present value terms.
6. Because  $k_0$  falls by Moore's rate, and assuming that the distribution of potential work is skewed such that there is more lower-valued than higher-valued work, the demand for computational power will grow exponentially over time.

For computation supplied via an intermediary:

7. The intermediary that charges for power on a per-hour-used basis must charge at a rate greater than the square of the power consumed otherwise the consumer has incentive to purchase infinite power.
8. Under pricing models described in Equation 7, the consumer may purchase less than  $rW$  gigaflops of power (but not less than  $rW/2$ ). This result highlights a market segment that computational intermediaries can serve and which cluster computers cannot. Further, because the lower limit on the power the consumer would demand from a cluster is a positive function of work, the size of the intermediary's market segment increases as the size of customers' work increases.

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